Energy Prices and Household Heterogeneity: Monetary Policy in a Gas-TANK*

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Abstract

How does household heterogeneity affect the transmission of an energy price shock? What are the implications for monetary policy? We develop a small, open-economy TANK model that features labor and an energy import good as complementary production inputs (Gas-TANK). Given such complementarities, higher energy prices reduce the labor share of total income. Due to borrowing constraints, this translates into a drop in aggregate demand. Higher price flexibility insures firm profits from adverse energy price shocks, further depressing labor income and demand. We illustrate how the transmission of shocks in a RANK versus a TANK depends on the degree of complementarity between energy and labor in production and the degree of price rigidities. Optimal monetary policy is less contractionary in a TANK and can even be expansionary when credit constraints are severe. Finally, the contractionary effect of an energy price shock on demand cannot be generalized to alternate supply shocks, as the specific nature of the supply shock affects how resources are redistributed in the economy.

JEL Codes: F41, E52, E31, E21, E23
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1 Introduction

In early 2022, energy prices rose to historically high levels as Russia's invasion of Ukraine increased the risk of disruptions to the energy trade (Figure 1). From the standpoint of an energy importer such as the UK or the EA, the developments in global energy prices represent a deterioration in the terms of trade. This implies a contraction in income flowing to domestic production inputs, including labor income. If households face limits in their access to financial markets, the contraction in income can translate into a drop in aggregate demand. An energy price shock, typically modeled as a supply shock, can therefore have a direct effect on aggregate demand.

The view that energy price shocks can have a demand-side impact is commonplace. Nevertheless, existing theoretical models do not capture such an effect.[1,2] We posit two features that are crucial for this link. First, and in line with models that study the macroeconomic effects of energy price shocks, our small open-economy model features a constant elasticity of substitution (CES) production technology with low elasticity of substitution between labor and imported energy (Antras, 2004; Chrinko, 2008; Karabarbounis and Neiman, 2014; Cantore et al., 2015; Hyun et al., 2022; Bachmann et al., 2022). The second key feature is household heterogeneity. We highlight the demand-side effects of this supply shock in a two-agent New Keynesian (TANK) model where agents differ in their sources of income and access to financial markets.[3] The model features two types of households: *constrained hand-to-mouth (HtM) or worker households*, who consume only out of their labor income and have no access to financial markets, and *unconstrained households*, who earn firm profits in addition to labor income and have free access to financial markets.

We show that the impact of energy prices on aggregate demand depends critically on the elasticity of substitution between production inputs and household heterogeneity. This is because the degree of substitutability among production inputs determines the response of households' income to the shock. In particular, assuming production inputs are reasonably difficult to substitute, an increase in the price of energy leads to a fall in the labour share of firms' expenditures.[4] As households differ in their access to borrowing and sources of income, a reduction in the labor share has a negative impact on aggregate demand for two reasons. First, it implies a reduction in the flow of income accruing to domestic factors of production. Due to credit constraints faced by a share of households, this translates into lower demand. Second, as constrained worker households rely more heavily on labor income, a lower labor share implies a redistribution of income against agents with a high marginal propensity to consume, which further depresses aggregate demand. This supply shock therefore has a self-correcting effect, as the consequent contraction in aggregate demand dampens inflationary pressures. To our knowledge,

[1]Among policymakers, an increase in energy prices is thought to slow economic growth through lower real incomes (Lane, 2022; Tenreyro, 2022; Broadbent, 2022; Schnabel, 2022). There is also evidence that firms perceive energy price shocks as shocks to demand (Lee and Ni, 2002). Hamilton (2008) shows that energy price shocks mainly affect the economy through a disruption in consumers' and firms' spending on non-energy goods and services.

[2]Models that feature an aggregate demand channel attribute the contraction in economic activity to the monetary policy response to higher inflation induced by the energy price shock (Medina and Soto, 2005). Blanchard and Galí (2007) show that real distortions can interact with shocks, creating non-trivial tradeoffs for policymakers as the gap between natural and efficient output is endogenous in response to supply shocks. Since there is no household heterogeneity and production is Cobb-Douglas, increases in energy prices would lead to an expansion in economic activity. Subsequent work has shown that this gap is larger when there is low substitutability between labor and energy in production Montoro (2012) and between energy and other goods in consumption (Natal, 2012).

[3]To be precise, we use the term "demand-side effects" to refer to the transmission of shocks via the dynamic IS equation in our model. Supply shocks with demand-side effects can also be found in models with complementarities between consumption goods and services (Corsetti et al., 2008) and complementarities among sectors (Guerrieri et al., 2022a; Cesa-Bianchi and Ferrero, 2021).

[4]Note that for Cobb-Douglas production technology (unitary elasticity of substitution), energy prices have no impact on the labor share of total factor expenditure.

this is the first paper to study this transmission channel alongside its implications for optimal monetary policy.

Compared to the representative household in a RANK (representative agent New Keynesian) model, the constrained worker household will experience a stronger consumption response to the real income squeeze following an energy price shock because of its inability to smooth consumption by borrowing. The channels we highlight are absent in the standard RANK model, which assumes all households are the same and that they can borrow to smooth consumption in the presence of adverse shocks.[5,6]

The magnitude of this demand-side channel also depends on the degree of price rigidities, as the aforementioned contraction in aggregate demand can be moderated by the behavior of markups. Given price rigidities, an increase in energy prices reduces firms' markups. This redistributes income in favor of constrained worker households, hence increasing aggregate demand. Instead, with higher price flexibility, firms are able to pass the cost of the more expensive energy to the workers by raising prices.

Is the demand contraction following an increase in energy prices a common feature of supply disturbances? To the contrary, we show that an energy price shock is unique in how it redistributes resources in the economy. Consider the dynamics following a productivity shock in our TANK model.[7] Both an increase in energy prices and an adverse productivity shock raise firms' marginal costs, leading to an increase in inflation. While the supply-side impact is the same, energy prices and productivity shocks yield opposite effects on the demand side. An adverse productivity shock leads to a fall in markups, as firms must hire more labor for the same amount of output. This increases constrained worker households' income, which boosts aggregate demand. However, an energy price shock in our model lowers constrained worker households' income and leads to a fall in economic activity.

An energy price shock also differs from a shock to markups. While both shocks depress aggregate demand, the underlying cause is not the same. Higher markups imply an increase in the profit share relative to the labor share of income. The redistribution of resources against the constrained worker households depresses aggregate demand. In the case of a markup shock, the drop in demand is therefore fully explained by an uneven impact of the shock on households' income, due to the unequal income composition between constrained worker households and unconstrained firm-owning households. In contrast, the demand effect following an energy price shock is largely explained by an unequal access to international credit markets.

The open-economy dimension is therefore crucial for explaining the dynamics in response to an energy price shock.[8] As standard in the TANK literature, amplification in our model depends on the shock

[5]The model nests an open-economy RANK (representative agent New Keynesian) model, incorporating the usual channels through which a terms of trade shock affects aggregate demand. On the one hand, an increase in energy prices expands economic activity through a higher relative price of energy, which leads to substitution from imported energy towards domestic labor. On the other hand, the endogenous monetary policy response to the inflationary pressure caused by the energy price shock contracts economic activity. A terms of trade effect is also operative, as higher real interest rates lead to a fall in exports due to an exchange rate appreciation. This interest rate channel captures the usual mechanism through which supply shocks depress economic activity in an open-economy RANK model (Bernanke et al., 1997; Leduc and Sill, 2004; Miyamoto et al., 2023). In other words, an energy price shock itself is not contractionary in a RANK model. However, energy price shocks do have a direct effect on aggregate demand in an open-economy TANK model.

[6]A sizable literature studies optimal monetary policy in two-country models with incomplete financial markets (Devereux and Sutherland, 2008; Benigno, 2009; Rabitsch, 2012; Farhi and Werning, 2016; De Paoli, 2009; Fanelli, 2019; Senay and Sutherland, 2016; Corsetti et al., 2010, 2022). We consider optimal monetary policy in an open-economy model with heterogeneous agents within countries.

[7]An energy price shock has also traditionally been modeled as a technology shock, or a shock that affects the productive capacity of the economy (Bruno and Sachs (1985), see Kilian (2008) for references). Kilian (2008) notes that such approaches are unable to explain large fluctuations in real output.

[8]Alongside Chen et al. (2023); Motyovszki (2020, 2023); Camara (2022), this is one of the first open-economy TANK models in the literature. The open-economy dimension of heterogenous agent models has been explored in Auclert et al. (2021); de Ferra et al. (2020); Cugat (2019). See Pieroni (2022) for the transmission of an energy price shock in a closed economy HANK model and Harrison et al. (2011) for the transmission channels of a permanent energy price shock in a RANK model.

affecting constrained households by more relative to the unconstrained households. However, in our open-economy TANK model, the variable which captures the relative impact of the shock is the consumption gap, defined as the difference between unconstrained and constrained household consumption, rather than the income gap. These two variables differ since unconstrained worker households can smooth consumption by borrowing from abroad. The cyclicality of the consumption gap therefore determines the amplification of shocks in an open-economy TANK model.[9]

Next, we consider a normative question: what is the optimal response of monetary policy to an energy price shock in our model and how does it depend on the degree of household heterogeneity? In contrast to a RANK economy, energy price shocks in the TANK economy have both supply and demand-side effects. On the one hand, higher energy prices place upward pressure on inflation, which calls for a monetary policy tightening. On the other hand, it restricts aggregate demand, which instead calls for a monetary loosening. In our baseline calibration, we find that in both the RANK and the TANK models, optimal monetary policy is contractionary in order to counteract the inflationary effect of the shock. However, in the TANK model, the negative impact of higher energy prices on aggregate demand mitigates inflationary pressures. An energy price shock therefore has a weaker inflationary effect, which requires a milder increase in the interest rate.[10] Finally, we explore conditions under which optimal policy may actually be expansionary in the presence of an adverse supply shock. We find that this is true when the share of financially constrained worker households is sufficiently high.[11]

To sharpen our results, the baseline scenario considers energy solely as a production input. We therefore follow the conventional approach of modeling an energy price shock as a shock that constrains productive capacity, yet we show that it still affects aggregate demand directly. Moreover, we find that an energy price shock can be regressive even though we abstract from features that would suggest such an effect. Specifically, we assume that both types of households are identical in labor supply and wages received.[12]

We also consider the other extreme, where energy is only a component of households' consumption baskets. In this extension, and analogous to the baseline case, the effects of an energy price shock are contingent on the elasticity of substitution between energy and domestically produced goods. Due to complementarities, the higher cost of energy leads to a reduction in the share of domestic goods in households' spending. As less resources are devoted to the purchase of domestically produced goods, households' income falls. While unconstrained worker households can maintain their consumption levels by borrowing from the foreign sector, constrained worker households must reduce their consumption, causing inequality to rise and aggregate demand to decline. We find that the energy price shock is still regressive, leading to a contraction in aggregate demand even when energy comprises an equal proportion of both constrained and unconstrained worker households' consumption baskets.[13]

[9]We also contribute to literature that studies the propagation of energy price shocks, in particular, the sizable impact such shocks have on economic activity despite being a small share of production (Kim and Loungani, 1992; Finn, 2000). Gelain and Lorusso (2022) provide evidence for a significant financial accelerator mechanism that amplifies the effects of an oil price shock on the US economy.

[10]Recent work by Guerrieri et al. (2022b) and Caballero and Simsek (2022) also provides conditions under which optimal monetary policy is less contractionary in response to supply shocks.

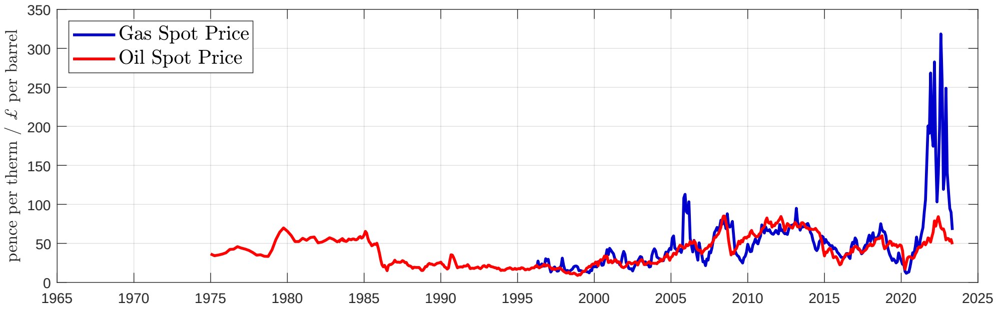
[11]Higher price flexibility also warrants more expansionary policy. The demand effect of higher energy prices depends on the evolution of firms' markups. If firms are able to increase prices to preserve markups, the costs of the energy price shock will be passed to workers, who will experience a more severe reduction in their income. Assuming a higher degree of price flexibility, constrained worker households experience a more pronounced drop in their income relative to unconstrained worker households, as reflected by the income gap. This leads to a deeper contraction in aggregate demand, which warrants looser monetary policy in the TANK model relative to its RANK counterpart.

[12]Känzig (2021) shows that carbon taxation imposes a larger burden on low-income households, since they are disproportionately employed in demand-sensitive sectors.

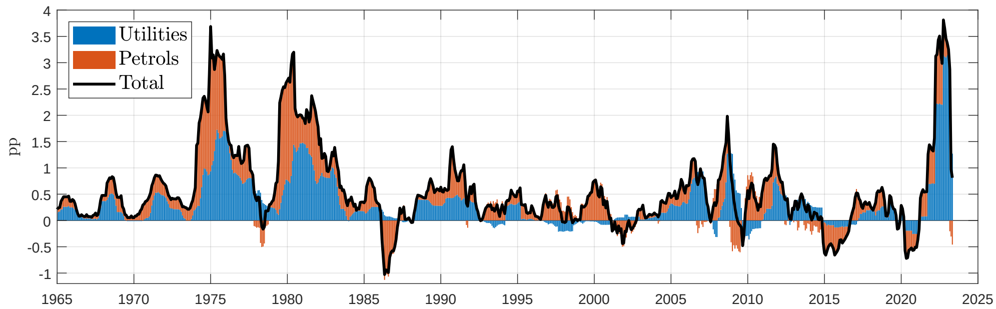
[13]Recent studies have noted the distributional impact of the energy price shock due to its effect on the consumption baskets

The main difference between the scenarios with energy in production versus consumption is the impact of energy price shocks on markups and the speed of transmission to aggregate demand. When energy is an input for firms, costlier energy transmits only gradually to the price of consumption goods, resulting in a decrease in markups. Therefore, profits partially absorb the effects of costlier energy, limiting the impact of the shock on the constrained worker households. However, when energy enters directly into the consumption basket, markups no longer absorb the shock. The shock therefore affects constrained worker households directly and to a greater degree, which exacerbates inequality.

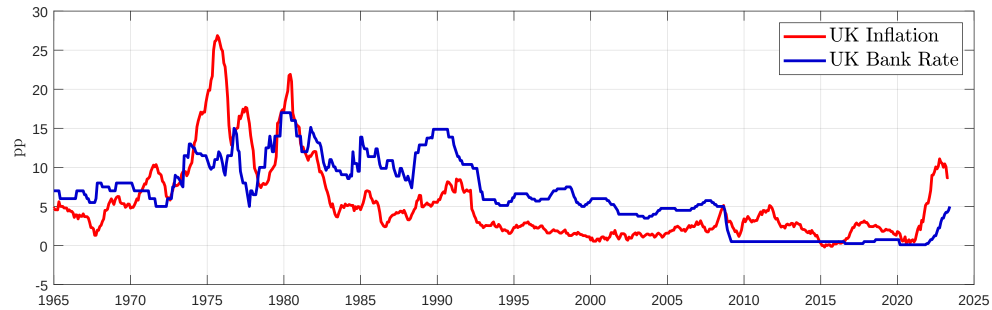
FIGURE 1: UK Energy Prices and Inflation



Notes: This panel shows the oil and gas spot prices for the UK, in £ per barrel and pence per therm, respectively. In mid-2022 the price of gas (blue line) had increased ten-fold, from an average of around 35 pence per therm before 2020 to a peak of around 350 pence per therm. Around the same time, the Sterling oil price (red line) reached an all-time high of 100£ per barrel.



Notes: A historical decomposition shows that these price increases have been a key driver of the high inflation rates that materialized in the UK in 2022. Almost 4 percentage points of the UK's 11% CPI inflation can directly be attributed to energy prices (blue bars). While the energy price shocks of the 1970s contributed to inflation mainly via increases in petrol prices, the shock of 2022 mainly contributed to inflation via an increase in utility prices.



Notes: We show the UK's CPI inflation and Bank Rate series. It is worth noting that inflation in the 1970s reached peaks above 20%, more than twice the peak of 2022/23, while the direct contribution of energy prices was broadly similar.

of heterogeneous households (Kuhn et al., 2021; Celasun et al., 2022; Bachmann et al., 2022; Battistini et al., 2022; Bhattarai et al., 2023; Peersman and Wauters, 2022; Bettarelli et al., 2023). An increase in energy prices can affect households' purchasing power through higher prices for energy products. Since poorer households spend a relatively large percentage of their income on energy, they receive a larger hit in terms of inflation when energy prices increase.

1.1 Related Literature

Our paper contributes to a literature that studies the transmission of shocks in heterogeneous agent models. The interaction of household heterogeneity with nominal rigidities can amplify the contractionary effect of TFP shocks on employment (Furlanetto and Seneca, 2012) and fiscal policy shocks on output (Galí et al., 2007a). However, we show that an interaction between household heterogeneity and production complementarity is crucial to generate the contractionary effect of an energy price shock on output. Our assumption of a CES production function with labor and energy allows for changes in energy prices to affect energy costs as a share of total income. More broadly, this paper builds on the vast literature that studies the implications of household heterogeneity for macroeconomic dynamics (Galí et al., 2007a; Bilbiie, 2008; Debortoli and Galí, 2017; Auclert et al., 2018; Kaplan and Violante, 2018; Bilbiie, 2019; Acharya and Dogra, 2020; Bilbiie, 2020; Broer et al., 2020; Bilbiie and Ragot, 2021; Cantore and Freund, 2021; Bilbiie et al., 2022). Challe et al. (2017) and Ravn and Sterk (2021) also show that a supply shock can have effects on the demand-side, but through a precautionary savings motive.

In concurrent and independent work, a closely related paper is Auclert et al. (2023) which differs from our approach primarily in its use of a HANK model instead of a TANK model. Although HANK models can capture more realistic distributional effects and shock propagation, TANK models aim to preserve tractability while matching the key features of HANK models.[14] This tractability facilitates our analysis of optimal monetary policy. Another key difference is that the two agents in our model differ in access to credit as well as income type, which allows us to consider the unequal incidence of the energy price shock on labor income versus profit income. As a result, the behavior and evolution of markups is important in our model (in addition to the degree of complementarity between energy and labor in production). We also show that in a TANK model, an energy price shock is unique among supply shocks in terms of its impact on the demand-side, due to the way in which resources are redistributed between constrained worker households and unconstrained households.

This paper also contributes to the growing literature that examines the macroeconomic implications of the recent surge in imported energy prices. Cardani et al. (2022); Hansen et al. (2023); Blanchard and Bernanke (2023); Gagliardone and Gertler (2023) decompose drivers of the large and persistent surge in inflation, as many countries faced a mix of supply and demand shocks due to commodity prices, monetary and fiscal policy, and constraints in goods and labor markets. Benigno and Eggertsson (2023) provide empirical evidence of significant nonlinearities and propose a New Keynesian model with search and matching frictions and wage rigidity to explain the rise in inflation. Recent studies have considered the welfare effects of the energy price shock. Del Canto et al. (2023) study the distributional consequences of inflationary oil shocks and monetary expansions in the US. Sterk et al. (2023) consider the positive and as normative implications of aggregate and sector-level shocks in a multi-sector New Keynesian model with non-homothetic preferences, and heterogeneity in income, wealth and consumption baskets. Given the distributional effects of an energy price shock, a number of studies have examined the role of fiscal policy. Motyovszki (2023) investigates the fiscal implications of an adverse terms-of-trade shock. A robust result is an increasing debt-to-GDP ratio as real growth slows due to a loss of domestic purchasing power and widening budget deficits, particularly if monetary policy tightens aggressively

[14] Heterogeneous-agent New Keynesian (HANK) models capture important distributional effects of macroeconomic policies, generating more realistic impulse response functions than traditional macroeconomic models. They can provide insights into the channels through which aggregate shocks propagate through the economy, which can inform the design of more effective policy responses. However, their complexity makes it difficult to study optimal policy. In contrast, two-agent New Keynesian (TANK) models offer a more analytically tractable framework that provides intuition for the underlying mechanisms at work. Recent research by Debortoli and Galí (2017) has shown that TANK models can match the key features of HANK models and produce consistent micro data and macroeconomic predictions (Bilbiie, 2008; Cantore and Freund, 2021).

and debt has short maturities. Kharroubi and Smets (2023) study the optimal fiscal policy response to energy price shocks in a model with household heterogeneity and non-homothetic preferences. Finally, Gornemann et al. (2022) study conditions under which supply constraints and energy shortages can raise the risk of self-fulfilling fluctuations. The presence of high-MPC households in their model warrants a more aggressive monetary policy response in order to guarantee equilibrium determinacy.

Roadmap The rest of this paper is structured as follows. In Section 2, we present our model with an emphasis on the key features: household heterogeneity and product input complementarity. We show how these features allow for demand-side effects of an energy price shock in Section 2.4. Section 3 presents the baseline calibration and impulse response functions, which illustrates the transmission channels we discuss. We show how the magnitude of the various channels depend on the severity of credit constraints, price rigidities and the degree of substitutability between production inputs. In Section 4, we compare the dynamics of an energy price shock to alternate supply shocks. We consider optimal monetary policy in Section 5. Section 6 explores an extension with energy as a consumption good. Section 7 concludes the paper.

2 Baseline Model

We begin our discussion of the baseline model with a focus on two key model features: household heterogeneity and imported energy as a complementary input to production.[15]

2.1 Household Heterogeneity

Unconstrained Households A fraction $(1-\omega)$ of households are financially *unconstrained* (denoted by u). They consume $C_{u,t}$, supply labor $N^h_{u,t}$ to unions, trade in domestic (foreign) nominal riskless bonds $B_{u,t}$ ($B^*_{u,t}$), and receive profits from firm ownership $DIV^F_{u,t}$. Their lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{u,t}^{1-\sigma}-1}{1-\sigma} - \chi \frac{(N^h_{u,t})^{1+\varphi}}{1+\varphi} \right).$$

Unconstrained households maximize their lifetime utility subject to their budget constraint

$$W^h_t N^h_{u,t} + R_{t-1}B_{u,t-1} + \mathcal{E}_t \bar{R}^* B^*_{u,t-1} + DIV^F_{u,t} + DIV^L_{u,t} = P_t C_{u,t} + B_{u,t} + \mathcal{E}_t B^*_{u,t} + T_{u,t} + P_t \mathcal{T}_u, \tag{2.1}$$

where R_{t-1} ($\bar{R}^*$) denotes the gross nominal rate of return on domestic (foreign) bonds, P_t is the price of the consumption good,[16] $\mathcal{E}_t$ is the nominal exchange rate (in domestic relative to foreign currency terms), $DIV^F_{u,t}$ represents profits derived from firm ownership, $DIV^L_{u,t}$ are profits transferred to the household by labor unions and $T_{u,t}$ are lump-sum transfers. $\mathcal{T}_u$ is a steady-state transfer from unconstrained to constrained households. The unconstrained household's consumption-savings Euler equation is

$$1 = \mathbb{E}_t \left[\Lambda_{u,t,t+1} \frac{R_t}{\Pi_{t+1}} \right], \tag{2.2}$$

[15] In section 6, we consider the case where energy is solely a consumption input. The full model derivation can be found in Appendix A.

[16] We assume that households' consumption basket only consists of the domestically produced final output good.

where $\Pi_t \equiv P_t/P_{t-1}$, $\Lambda_{u,t,t+1} \equiv \beta\left(C_{u,t}/C_{u,t+1}\right)^{\sigma}$ and the UIP condition is given by

$$0 \quad = \quad \mathbb{E}_t\left[\Lambda_{u,t,t+1}\frac{1}{\Pi_{t+1}}\left(R_t - \bar{R}^*\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)\right]. \tag{2.3}$$

Constrained Households The remaining fraction ω of households are *financially constrained* (denoted by c) 'hand-to-mouth' households. They only receive labor income, hence their consumption is

$$P_t C_{c,t} = W_t^h N_{c,t}^h + DIV_{c,t}^L - T_{c,t} + P_t\mathcal{T}_c. \tag{2.4}$$

The wage received by households W_t^h is determined as a function of a weighted average of unconstrained and constrained households' marginal rate of substitution. Furthermore, we assume firms distribute labor demand equally among households, so that $N_{u,t}^h = N_{c,t}^h$. Aggregate consumption is

$$C_t = (1-\omega)C_{u,t} + \omega C_{c,t}. \tag{2.5}$$

We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}. \tag{2.6}$$

2.2 Production Input Complementarity

Final output production involves perfectly competitive final good packers and monopolistically competitive final good producers.

Final good packers Final good packers operate in a competitive market and produce the aggregate final good Z_t by combining a continuum of varieties $Z_t(i)$ with measure one, $Z_t = \left(\int_0^1 (Z_t(i))^{\frac{\epsilon_z - 1}{\epsilon_z}} di\right)^{\frac{\epsilon_z}{\epsilon_z - 1}}$. Optimization implies the following demand function for variety i, $Z_t(i) = (P_t(i)/P_t)^{-\epsilon_z} Z_t$, where $P_t \equiv \left(\int_0^1 (P_t(i))^{1-\epsilon_z} di\right)^{\frac{1}{1-\epsilon_z}}$ is the price of the final composite good. It can be shown that $P_t Z_t = \int_0^1 P_t(i) Z_t(i) di$.

Final good producers A continuum of final output producing firms, indexed by $i \in [0,1]$, operate in a monopolistically competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. Firm i produces the final output variety $Z_t(i)$ using the following CES production technology with imported energy ($E_t^z(i)$) and labor ($N_t(i)$) as inputs

$$Z_t(i) = \varepsilon_t^{TFP}\left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}(N_t(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}}(E_t^z(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}}\right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}, \tag{2.7}$$

where ε_t^{TFP} represents productivity and ψ_{ez} is the elasticity of substitution between energy and labor. The firm purchases labour from a union, paying the nominal wage W_t and it purchases energy from an importer at the nominal domestic currency price P_t^E.

Cost Minimization The final output producer's factor demand schedules are given by

$$W_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{MC_t^Z}{\tau_t^Z}\left(\frac{Z_t(i)}{N_t(i)}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}, \qquad P_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}}\frac{MC_t^Z}{\tau_t^Z}\left(\frac{Z_t(i)}{E_t^z(i)}\right)^{\frac{1}{\psi_{ez}}}\left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}},$$

where the Lagrange multiplier $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, i.e. the nominal marginal cost, and $\tau_t^Z = \tau^Z \varepsilon_t^{\mathcal{M}_z}$ is a shock to final output marginal costs that is isomorphic to a price markup shock.

Price Setting Firms face price stickiness à la Calvo, resetting prices in every period with probability $(1-\phi_z)$. A firm that is able to reset prices in period t chooses the price $P_t^\#$ that maximizes the sum of discounted profits subject to the demand faced in $t+s$

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\phi_z)^s \{\Lambda_{u,t,t+s}(P_t^\# Z_{t+s|t} - MC_{t+s}^Z Z_{t+s|t})\} \quad s.t. \quad Z_{t+s|t} = \left(\frac{P_t^\#}{P_{t+s}}\right)^{-\epsilon_z} Z_{t+s}.$$

Profit maximization implies $\mathbb{E}_t \sum_{s=0}^{\infty} (\phi_z)^s \{\Lambda_{u,t,t+s} Z_{t+s|t}(P_t^\# - \mathcal{M}_z MC_{t+s|t}^Z)\} = 0$, where $\mathcal{M}_z \equiv \frac{\epsilon_z}{\epsilon_z - 1}$ is the desired final output price markup.

2.3 Remaining Features

Wage Stickiness As described in detail in Appendix (A.2), we incorporate wage stickiness following Schmitt-Grohe and Uribe (2006). This gives rise to a standard wage inflation equation.

Imports Each energy import good that the final output good producer demands is supplied by a perfectly competitive energy importer.[17] Energy importers buy energy on the world market from foreign energy exporters at foreign currency energy price $P_t^{E,*}$ and sell it to domestic final output producers

$$p_t^E = \mathcal{Q}_t p_t^{E,*}. \tag{2.8}$$

We define the real exchange rate $\mathcal{Q}_t \equiv \mathcal{E}_t P_t^* / P_t$ and the real foreign and domestic energy price definition $p_t^{E,*} \equiv P_t^{E,*}/P_t^*$ and $p_t^E \equiv P_t^E / P_t$. We model the foreign currency energy price level $p_t^{E,*}$ as an exogenous AR(1) process in which ε_t^E denotes an energy price shock process

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E} \left(p_{t-1}^{E,*}\right)^{\rho_E} \varepsilon_t^E. \tag{2.9}$$

Exports The global demand schedule for the bundle of domestic non-energy exports X_t depends on the foreign currency price of domestic non-energy exports[18], $P_t^{EXP} = P_t / \mathcal{E}_t$, relative to the world non-energy export price, P_{ss}^{X*}, and the world trade volume Y_{ss}^*

$$X_t \quad = \quad \kappa^* \left(\frac{P_t^{EXP}}{P_{ss}^{X*}}\right)^{-\varsigma^*} Y_{ss}^*. \tag{2.10}$$

ς^* is the substitution elasticity between differentiated non-energy exports in the rest of the world.

Retailers Perfectly competitive retailers buy final output goods from the final output packers at price P_t and convert them into domestic consumption goods and export goods.

[17] We abstract from non-energy imports.

[18] We assume that energy cannot be produced domestically and hence cannot be exported.

Monetary policy The central bank follows a Taylor rule that responds to deviations of (annual) inflation[19] and the output gap from their targets,

$$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^a}{\bar{\Pi}^a} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\tilde{Y}_t \right)^{(1-\theta_R)\theta_Y}.$$

The output gap is defined as the ratio of employment to flexible price/wage employment, $\tilde{Y}_t = N_t / N_t^{flex}$.

Shock Processes The model features three shocks: (i) a TFP shock η_t^{TFP}, (ii) a price markup shock $\eta_t^{\mathcal{M}_z}$ and (iii) an energy price shock η_t^E

$$\begin{aligned}
\log \varepsilon_t^{TFP} &= \rho_{TFP} \log \varepsilon_{t-1}^{TFP} - \varsigma_{TFP} \eta_t^{TFP}, \quad && \eta_t^{TFP} \sim N(0,1) \\
\log \varepsilon_t^{\mathcal{M}_z} &= \rho_{\mathcal{M}_z} \log \varepsilon_{t-1}^{\mathcal{M}_z} - \varsigma_{\mathcal{M}_z} \eta_t^{\mathcal{M}_z}, \quad && \eta_t^{\mathcal{M}_z} \sim N(0,1) \\
\log \varepsilon_t^{E} &= \varsigma_E \eta_t^E, && \eta_t^E \sim N(0,1).
\end{aligned}$$

where ρ_i and ς_i, $i \in \{TFP, \mathcal{M}_z, E\}$ denote the degrees of persistence and the standard deviations of the shock processes. We abstract from demand shocks, since the focus of our paper is on the transmission of 'supply' shocks and their 'demand-side' effects.

2.4 Log-linearised Model Summary

Introducing lower case variables with a hat superscript to refer to log-deviations from steady state, i.e. $\hat{x}_t = \log(x_t/x_{ss})$, we can summarize the key equations of the log-linearised model as follows.

Household Demand The aggregate consumption Euler equation (2.11) features the consumption gap $\hat{\gamma}_t$. We will show below how energy shocks transmit via $\hat{\gamma}_t$ and hence affect aggregate demand

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega \Delta \hat{\gamma}_{t+1}] - \frac{1}{\sigma}(\hat{r}_t - \mathbf{E}_t[\hat{\pi}_{t+1}]) \tag{2.11}$$

$$\hat{\gamma}_t \equiv \hat{c}_{u,t} - \hat{c}_{c,t}, \quad \hat{c}_t = \omega \hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t}. \tag{2.12}$$

The budget constraints (2.1) and (2.4) that pin down $\hat{c}_{u,t}$ and $\hat{c}_{c,t}$ and hence the sources of income of unconstrained and constrained households will be crucial determinants of $\hat{\gamma}_t$.

Wage and Price Setting Nominal wage and price stickiness gives rise to standard wage and price inflation equations (2.13) and (2.14). The log-linearised factor demand schedules are given by (2.15) and (2.16). Note that the time varying price markup is the inverse of the marginal cost, $\hat{\mu}_t^Z = -\hat{mc}_t^Z$.

$$\hat{\pi}_t^W = \frac{(1-\phi_w\beta)(1-\phi_w)}{\phi_w}\left(\hat{w}_t^h - \hat{w}_t\right) + \beta E_t\left[\hat{\pi}_{t+1}^W\right], \quad \hat{\pi}_t^W = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{2.13}$$

$$\hat{\pi}_t = \frac{(1-\phi_z\beta)(1-\phi_z)}{\phi_z}\left(\hat{mc}_t^Z\right) + \beta E_t[\hat{\pi}_{t+1}] \tag{2.14}$$

$$\hat{mc}_t^Z = \hat{w}_t + \hat{\varepsilon}_t^{\mathcal{M}_z} - \psi_{ez}^{-1}(\alpha_{ez}\hat{e}_t^z - \alpha_{ez}\hat{n}_t) - \hat{\varepsilon}_t^{TFP} \tag{2.15}$$

$$\hat{mc}_t^Z = \hat{p}_t^E + \hat{\varepsilon}_t^{\mathcal{M}_z} - \psi_{ez}^{-1}((1-\alpha_{ez})\hat{n}_t - (1-\alpha_{ez})\hat{e}_t^z) - \hat{\varepsilon}_t^{TFP} \tag{2.16}$$

[19]Note that in the case in which energy does not enter the consumption basket, domestic final output price 'core' inflation is equal to CPI inflation, $\Pi_t^a = \Pi_t^{CPI,a}$. We will relax the assumption around energy entering the consumption basket in Section 6.

Market Clearing and Monetary Policy Goods market clearing is given by equation (2.17) and implies that final output $\hat{z}_t$ is split between domestic consumption and exports

$$\hat{c}_t = \frac{Z_{ss}}{C_{ss}}\hat{z}_t - \frac{X_{ss}}{C_{ss}}\hat{x}_t, \quad \hat{z}_t = \hat{\varepsilon}_t^{TFP} + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}_t^z \tag{2.17}$$

$$\hat{r}_t = \theta_R \hat{r}_{t-1} + (1-\theta_R)\left((\theta_\pi/4)\hat{\pi}_t^{CPI,a} + \theta_Y(\hat{n}_t - \hat{n}_t^{flex})\right), \quad \hat{\pi}_t^{CPI,a} \equiv \sum_{j=0}^{3}\hat{\pi}_{t-j}^{CPI}, \quad \hat{\pi}_t^{CPI} = \hat{\pi}_t \tag{2.18}$$

where the log-deviation of exports from steady state is $\hat{x}_t = \varsigma^* \hat{q}_t$, as implied by Equation (2.10) and domestic energy prices are $\hat{p}_t^E = \hat{p}_t^{E,*} + \hat{q}_t$. As outlined above, $\hat{p}_t^{E,*}$ follows an exogenous AR(1) process and the real exchange rate $\hat{q}_t$ is pinned down by a UIP condition $\hat{q}_t = \mathbf{E}_t\hat{q}_{t+1} - (\hat{r}_t - \mathbf{E}_t\hat{\pi}_{t+1})$. Monetary policy sets the nominal interest rate $\hat{r}_t$ and follows a Taylor rule.

Demand-side Effects of Energy Price Shocks Taking total employment $\hat{n}_t$ as a measure of domestic real activity and a proxy for value added output (GDP), we can use (2.11), (2.12), (2.15), (2.16) and (2.17) to derive the following dynamic IS equation (see Appendix A.9.1 for details)

$$\begin{aligned}\hat{n}_t \quad = \quad & \mathbf{E}_t[\hat{n}_{t+1}] - \frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}(\hat{r}_t - \mathbf{E}_t[\hat{\pi}_{t+1}]) + \omega\frac{C_{ss}}{Z_{ss}}\mathbf{E}_t[\Delta\hat{\gamma}_{t+1}] - \frac{X_{ss}}{Z_{ss}}\varsigma^*\mathbf{E}_t\Delta\hat{q}_{t+1} \\ & -\psi_{ez}\alpha_{ez}\left(\mathbf{E}_t\Delta\hat{p}_{t+1}^E - \mathbf{E}_t\Delta\hat{w}_{t+1}\right) + \mathbf{E}_t\left[\Delta\hat{\varepsilon}_{t+1}^{TFP}\right].\end{aligned} \tag{2.19}$$

Solving this expression forward we obtain a dynamic IS curve that breaks down the channels through which energy prices affect activity

$$\hat{n}_t = \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}(\hat{r}_{t+k} - \hat{\pi}_{t+k+1})}_{\text{Inter-temporal substitution (-)}} \underbrace{-\omega\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\substack{\text{Demand effect from}\\ \text{credit constraints (+/-)}}} \underbrace{+\psi_{ez}\alpha_{ez}\left(\hat{p}_t^E - \hat{w}_t\right)}_{\substack{\text{Intra-temporal substitution}\\ \text{in production (+)}}} \underbrace{+\frac{X_{ss}}{Z_{ss}}\varsigma^*\hat{q}_t}_{\text{terms of trade (+)}} -\hat{\varepsilon}_t^{TFP}. \tag{2.20}$$

According to equation (2.20), economic activity $\hat{n}_t$ depends on the path of the real interest rate, the consumption gap $\hat{\gamma}_t$, the real price of energy $\hat{p}_t^E$ relative to the real wage $\hat{w}_t$ and on foreign demand for the domestic good, which is determined by the real exchange rate $\hat{q}_t$. An increase in the consumption gap $\hat{\gamma}_t$ in response to a shock reflects redistribution against the constrained workers. As constrained agents have a higher marginal propensity to consume, such redistribution causes a drop in aggregate demand that brings GDP down. The effect of the consumption gap on GDP is increasing in the share ω of constrained households.

The IS equation illustrates the channels through which an energy price shock affects economic activity. Since our model nests a RANK ($\omega = 0$), the channels present in a RANK are also present here. In a RANK, energy prices operate through two different channels.

First, an increase in energy prices stimulates GDP through a higher price of energy $\hat{p}_t^E$ relative to wages $\hat{w}_t$, which leads to substitution from imported energy towards the domestic labor input (intra-temporal substitution effect).

Second, given the inflationary pressures derived from the shock, the central bank responds by tightening monetary policy. The ensuing increase in the real rate contracts economic activity (inter-temporal substitution effect) and leads to a fall in exports due to an exchange rate appreciation (terms of trade effect). This interest rate channel captures the usual mechanism through which supply shocks depress economic activity in an open-economy RANK model. In contrast to our open-economy TANK model, the energy shock itself is not contractionary in a standard open-economy RANK model. The economic downturn in the RANK model is purely a result of the monetary policy response to inflation caused by the energy shock.

A new channel for supply shocks is present in the TANK economy, as indicated by the term involving the consumption gap. This term captures a demand-side impact of the energy shock that operates through an income effect. The sign of this demand-side effect depends on the response of the consumption gap to the shock. Under reasonable calibrations (where inputs are largely complements), an increase in the price of energy translates into a contraction in households' income, as more resources must be devoted to the purchase of the energy input. Given financial constraints, demand by worker households falls (as reflected by an increase in the consumption gap), leading to an economic recession.

Next, we discuss the aforementioned demand-side effect that emerges from credit constraints by analyzing how the energy price shock affects the consumption gap.

The Consumption Gap Letting $INC_{u,t}$ and $INC_{c,t}$ denote unconstrained and constrained households' current income, and using the budget constraints (2.4) and (2.1), we can decompose the consumption gap into an income gap $\Gamma_t^{inc} \equiv INC_{u,t}/INC_{c,t}$ and into a foreign borrowing component[20]

$$\Gamma_t = \Gamma_t^{inc} + \frac{\mathcal{E}_t(\bar{R}^* - 1)B^*_{u,t-1} - \mathcal{E}_t \Delta B^*_{u,t}}{INC_{c,t}}. \tag{2.21}$$

Equation (2.21) illustrates how an energy price shock affects the consumption gap through a differential impact on constrained and unconstrained households consumption. An unequal consumption response can have two sources. One source is through changes in the income gap, which reflects the different impact of the shock on current income (due to differences in income composition). The other source is access to borrowing (reflected in changes in foreign bond holdings $\Delta B^*_{u,t}$), which allows unconstrained households to insure their consumption from income fluctuations following an energy price shock.

Let's consider how these two components of the consumption gap are determined. From the economy's budget constraint, we know that the evolution of foreign bonds depends on the balance of trade. Therefore, the consumption gap can be rewritten as

$$\Gamma_t = \Gamma_t^{inc} - \frac{1}{1-\omega} \frac{TB_t}{INC_{c,t}}, \tag{2.22}$$

where $TB_t = P_t^X X_t - P_t^E E_t^z$ is the trade balance. Using the definitions of unconstrained and constrained total income, the above expression can be written as follows

$$\Gamma_t = 1 + \underbrace{\frac{1}{1-\omega} \frac{\mathcal{M}_t^Z - 1}{\Xi_t^N}}_{\text{Income gap}} + \underbrace{\frac{1}{1-\omega}\left(\frac{1}{\Xi_t^N} - 1 - \frac{P_t^X X_t}{INC_{c,t}}\right)}_{\text{Borrowing}}, \quad \frac{\partial \Gamma_t}{\partial \mathcal{M}_t^Z} > 0, \quad \frac{\partial \Gamma_t}{\partial \Xi_t^N} < 0 \tag{2.23}$$

where $\mathcal{M}_t^Z \equiv P_t / MC_t^Z$ is firms' average markup, $\Xi_t^N \equiv W_t N_t / (W_t N_t + P_t^E E_t^z)$ is the labor share of firms' total expenditure. Equation (2.23) indicates that the effect of a change in energy prices on the consumption gap (and hence, on aggregate demand) is determined by the impact of the shock on two key variables, firms' markups $\mathcal{M}_t^Z$ and the labor share Ξ_t^N.

The income gap (and hence, the consumption gap) depends positively on firms' markups, since an

[20]The expression for the consumption gap takes into account that domestic bonds must equal zero in equilibrium.

increase in the markup redistributes resources towards the unconstrained firm owners. The income gap also increases in response to a reduction of the labor share in total factor expenditure, since a reduction in the labor share redistributes resources against the constrained workers and towards the import of energy. A reduction in the labor share also increases the consumption gap due to unconstrained households' ability to insure their consumption by borrowing. The reduction in the labor share reflects an increase in the resources devoted to imported energy (an increase in the energy share), which must be financed via an increase in foreign debt. Borrowing from the foreign sector is used by unconstrained households to finance their consumption, hence increasing the consumption gap.

Finally, to understand how the energy price shock affects firms' average markup and the labor share, notice that these two objects are linked to the price of energy according to the following expressions[21]

$$\mathcal{M}_t^Z = \frac{\varepsilon_t^{TFP} P_t}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}} + \alpha_{ez}\left(P_t^E\right)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}}, \quad \frac{\partial \mathcal{M}_t^Z}{\partial P_t^E} < 0 \tag{2.24}$$

$$\Xi_t^N = \left(1 + \frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{P_t^E}{W_t}\right)^{1-\psi_{ez}}\right)^{-1}, \quad \frac{\partial \Xi_t^N}{\partial P_t^E} < 0. \tag{2.25}$$

Notice from (2.24) that, given price rigidities, an increase in energy prices (P_t^E) reduces firms' markups. This implies a redistribution of income in favor of workers, reflected in an reduction of the consumption gap (equation (2.23)). This boosts aggregate demand, and hence, activity (equation (2.20)).

Equation (2.25) shows that the impact of higher energy prices on the labor share crucially depends on the elasticity of substitution between energy and labor (ψ_{ez}). In the case of a Cobb-Douglas production technology ($\psi_{ez} = 1$) we have $\Xi_t^N = 1 - \alpha_{ez}$, implying that the price of energy has no impact on the labor share. If the elasticity of substitution is larger than one ($\psi_{ez} > 1$), higher energy prices increase the labor share. The reason is that costlier energy triggers a strong substitution from energy towards labor. The resulting redistribution of income in favor of workers is reflected in a reduction in the consumption gap, which boosts aggregate demand and activity. Alternatively, if energy cannot easily be substituted for by labor ($\psi_{ez} < 1$), an increase in energy prices reduces the labor share. The shock therefore redistributes against the constrained workers, increases the consumption gap and depresses economic activity.

In the next section, we cite empirical evidence that finds low substitutability between labor and energy. In this scenario, we should expect that an increase in energy prices will reduce both the labor share and firms markups (i.e., the profit share). The relative impact of the shock on these two objects will determine whether the constrained workers of firm owners are mostly affected, and hence, the size of the demand-side effect of the energy price shock.

3 Dynamic Responses under a Taylor Rule

Parameterization We list the calibration for key model parameters in Table 1. To stay close to the literature, we calibrate our model using some common parameterizations. We assume a discount factor, β, of 0.9994. The elasticities of substitution across goods varieties (ϵ_z) and across worker types (ϵ_w) are both set to 11, which implies a markup of 10% in steady state. We assume goods prices and wages are adjusted with Calvo parameters $\phi_z = 0.66$ and $\phi_w = 0.92$. We set the response to inflation (θ_π) and slack (θ_Y) in the Taylor rule to 1.5 and 0.125, respectively. The interest rate smoothing parameter (θ_R) is set to

[21] The expression for the labor share is obtained using firms' demand functions for energy and labor.

0.9. The productivity process parameters are set to $\rho_{TFP} = 0.93$ and $\varsigma_{TFP} = 0.07$. The energy price shock has persistence $\rho_E = 0.8$ and $\varsigma_E = 1$. In the experiments below, we will look at the case of an increase of energy prices by 100% on impact. The markup shock has persistence $\rho_{\mathcal{M}_z} = 0.9$ and $\varsigma_{\mathcal{M}_z} = 0.1$. The population share of constrained worker households (ω) is set to 0.25.

TABLE 1: PARAMETER VALUES

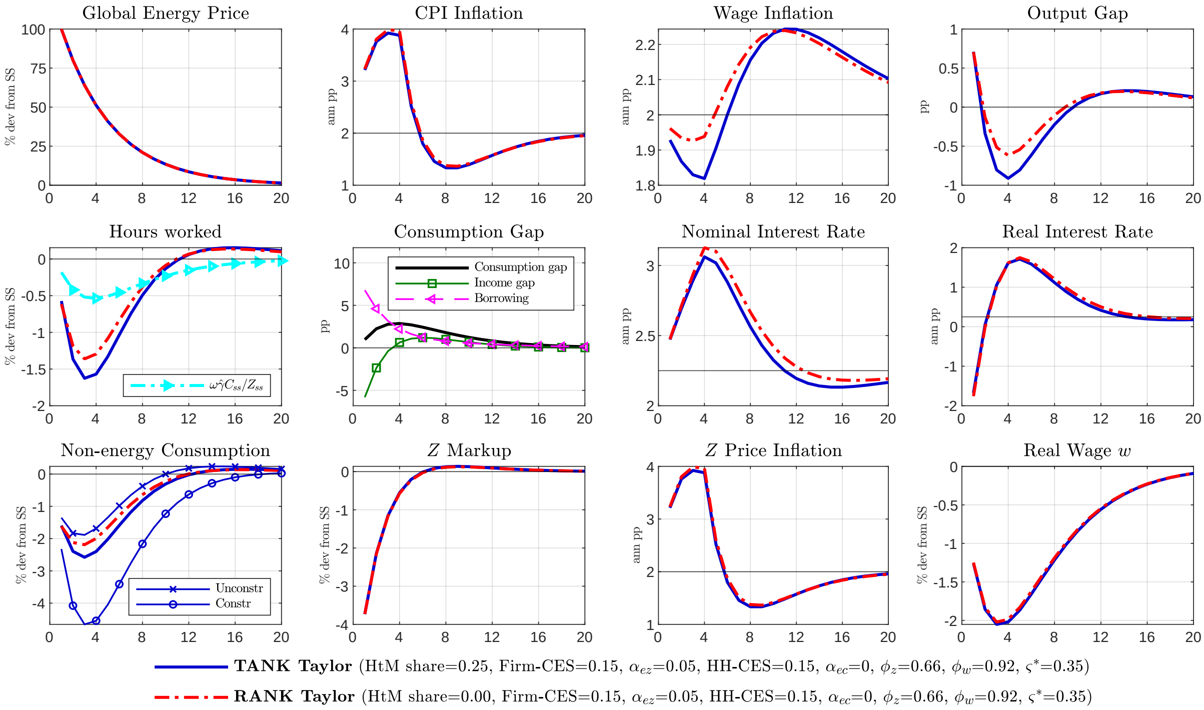
Parameter	Definition	Value	Source/Target
Households			
β	Household discount factor	0.9994	Annual net nominal rate $r_{ss} \approx 2.25\%$
σ	Household risk aversion	1.0000	Literature
χ	Utility weight of labour	1.4102	$L_{ss} = 1$
φ	Inverse Frisch elasticity	2	Literature
ω	Share of constrained households	0.2500	Literature
Labour Unions			
ϵ_w	Elasticity of substitution for labour	11.0000	10 % gross wage markup
ϕ_w	Calvo wage adjustment	0.9200	Schmitt-Grohe and Uribe (2006)
Firms			
α_{ez}	Energy share in production	0.05	5 % energy share in production
ψ_{ez}	CES degree btw energy and labour in production	0.15	UK estimates
ϵ_z	Elasticity of substitution for goods	11.0000	10 % gross final goods markup
ϕ_z	Calvo price adjustment	0.6600	Avg lifetime of prices 3Q
Monetary Policy			
θ_Π	Interest rate sensitivity to inflation	1.5000	Literature
θ_Y	Interest rate sensitivity to output	0.1250	Literature
θ_R	Interest rate smoothing	0.9000	Literature
$\bar{\Pi}$	Inflation target	1.0050	2% Target
World Trade			
κ^*	Foreign preference for domestic exports	0.2632	export share X_{ss}/Z_{ss}=0.25
ς^*	Price elasticity of world demand for dom. exports	0.35	
Shock Processes			
ρ_{TFP}	Persistence of TFP shock	0.93	Fernald 2014
$\rho_{\mathcal{M}_z}$	Persistence of price markup shock	0.9	
ρ_E	Persistence of global energy price shock	0.8	Fall of energy price by 50% after 4Q
ς_{TFP}	Standard deviation of TFP shock	0.07	Fernald 2014
$\varsigma_{\mathcal{M}_z}$	Standard deviation of price markup shock	0.1	
ς_E	Standard deviation of global energy price shock	1	10 st.dev. shock leads to 100% increase on impact

We limit our discussion to the following key parameters: the elasticity of substitution between energy and labor in production ($\psi_{ez} = 0.15$) and the steady state share of energy in production ($\alpha_{ez} = 0.05$).[22] There are a wide range of estimates for the elasticity of substitution between production inputs in the literature. Higher estimates, such as those provided by Bodenstein et al. (2012) (0.42) are motivated by estimates of the short-run price elasticity of oil demand from structural econometric models. Natal (2012) sets this parameter to 0.3, while Plante (2014) suggests a calibration of 0.25 so that the own price elasticity of oil is approximately -0.25. Montoro (2012) sets the value of the elasticity of substitution between oil and labor at 0.2, equal to the average value reported by Hamilton (2009). On the low end of estimates is Adjemian and Darracq Paries (2008) and Backus and Crucini (2000), at 0.09. However, their production function is Cobb-Douglas in labor and a capital services-energy mix, where the latter is combined via CES. Finally, Stevens (2015) suggests an elasticity of substitution between oil and value-added of 0.03, where value-added is a Cobb-Douglas function with labor and capital inputs. This parameter is equivalent to the short-run oil demand elasticity and is chosen to be consistent with reduced-form evidence on the slope of the oil demand curve that lie between 0 and 0.11. Between the extreme cases of zero and infinite substitutability, the effects of an energy price shock on macroeconomic aggregates also depends on the share of energy in production. The share of energy in production ranges from 2% in Natal (2012) for the US, 4% in Bachmann et al. (2022) for Germany, and 5% in Stevens (2015).

[22] In section 6 we will introduce an extension of the model in which we allow for energy in the consumption basket. For simplicity, we will chose the same energy share and the same degree of substitutability as discussed here, $\alpha_{ec} = 0.05$ and $\psi_{ec} = 0.15$.

Energy Price Shock IRFs In Figure 2, we show the response to an increase in energy prices in the baseline model outlined above. In the RANK economy (red lines), an energy price shock places upward pressure on production costs, leading to a surge in inflation.

FIGURE 2: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function. Full set in Figure B.2.

The central bank responds by tightening monetary policy, which induces a contraction in activity. Relative to the RANK, the TANK economy (blue lines) experiences a deeper contraction. Moreover, while the recession in the RANK originates from the contractionary policy implemented by the central bank, in the TANK it is driven, to a considerable degree, by the direct impact of higher energy prices on aggregate demand. Since production inputs are complementary in the TANK economy, higher energy prices reduce the labor share of total income, implying a drop in labor earnings. Given borrowing constraints for worker households, this translates into a fall in aggregate demand. The employment decomposition in Figure 2 shows how much the fall in demand as a result of credit frictions contributes to the contraction in employment in the TANK economy.[23] Due to the adverse effect of the energy price shock on demand, monetary policy in the TANK is looser. A more accommodative monetary policy limits the decline in employment in the TANK economy compared to the RANK economy. In the TANK, the more accommodative monetary policy counteracts the contractionary impact of energy prices resulting from credit constraints. If simulations were conditioned on the RANK-implied policy path of the real interest rate, the TANK economy would experience a deeper contraction in employment compared to the RANK economy.

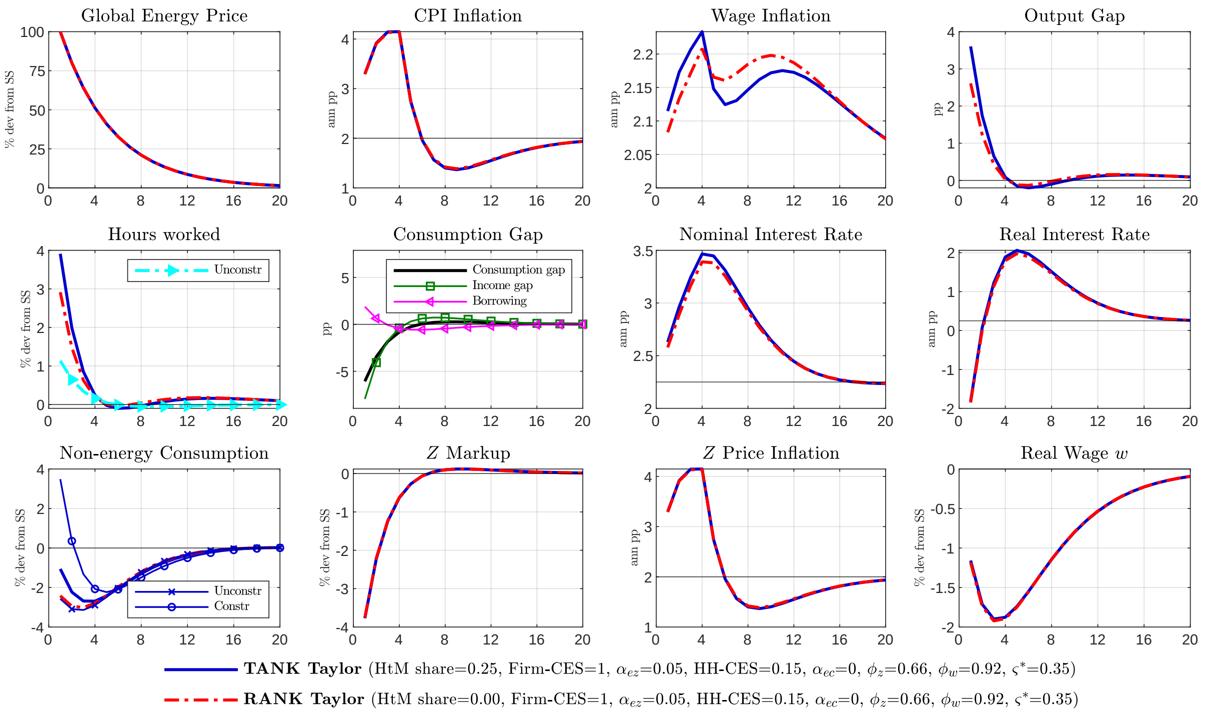
In the second row and second column panel of Figure 2 we show the dynamics of the consumption gap and its drivers (see equation (2.23)). The consumption gap fluctuates due to households' unequal income composition (green square line) and unequal access to credit (pink triangle line). While con-

[23] Equation 2.20 presents a decomposition for hours worked. The light blue dashed line represents the variation in hours worked due to the demand effect from credit constraints ($\hat{\gamma}_t$-term in equation 2.20).

sumption of constrained HtM households largely falls due to the drop in their income, unconstrained firm owner households are able to insure their consumption by borrowing from the external sector. The unequal access to borrowing (pink triangle line) explains the increase in the consumption gap. Initially, a drop in firms' markups (due to costlier energy) results in a negative income gap, which limits the increase in the consumption gap. Over time, as firms pass the costs of the shock to constrained HtM households through an increase in prices, markups recover and the income gap goes up as well, further raising the gap in consumption.

Role of complementarities Figure 3 illustrates the effects of the energy price shock under a higher degree of substitutability between labor and energy in production (Cobb-Douglas, $\psi_{ez} = 1$).

FIGURE 3: Dynamic Responses to a Global Energy Price Shock: Cobb-Douglas instead of CES Production Function

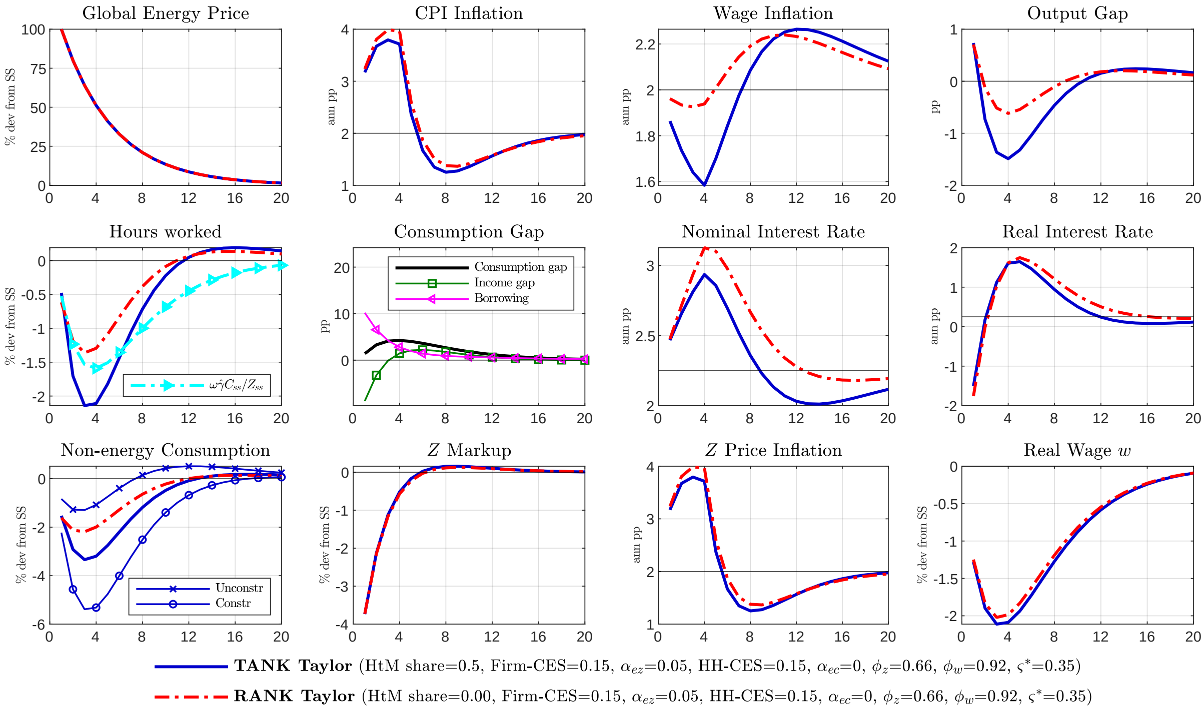


Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the production function. Full set in Figure B.16.

The increase in the price of energy imports imply a fall in the relative price of labor, which now leads to greater degree of substitution towards labor. The labor demand schedule shifts upwards and employment increases. For $\psi_{ez} = 1$, higher employment fully compensates for the lower relative wage, leaving the labor share constant. While the labor share remains constant, firms' markups experience a reduction due to higher marginal costs. Redistribution in favor of constrained HtM households, reflected in a reduction of the consumption gap, boosts aggregate demand. Given the positive effect of the shock on demand, the TANK economy experiences a milder recession relative to its RANK counterpart.

Higher share of constrained households Figure 4 illustrates the effects of the energy price shock when we assume a larger share of constrained households. Given more severe borrowing constraints, consumption becomes more responsive to the drop in households' income. It follows that the increase in energy prices induces a stronger fall in aggregate demand, leading to a deeper recession than in the baseline case (Figure 2).

FIGURE 4: Dynamic Responses to a Global Energy Price Shock: Higher Share of Constrained Households



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the production function.

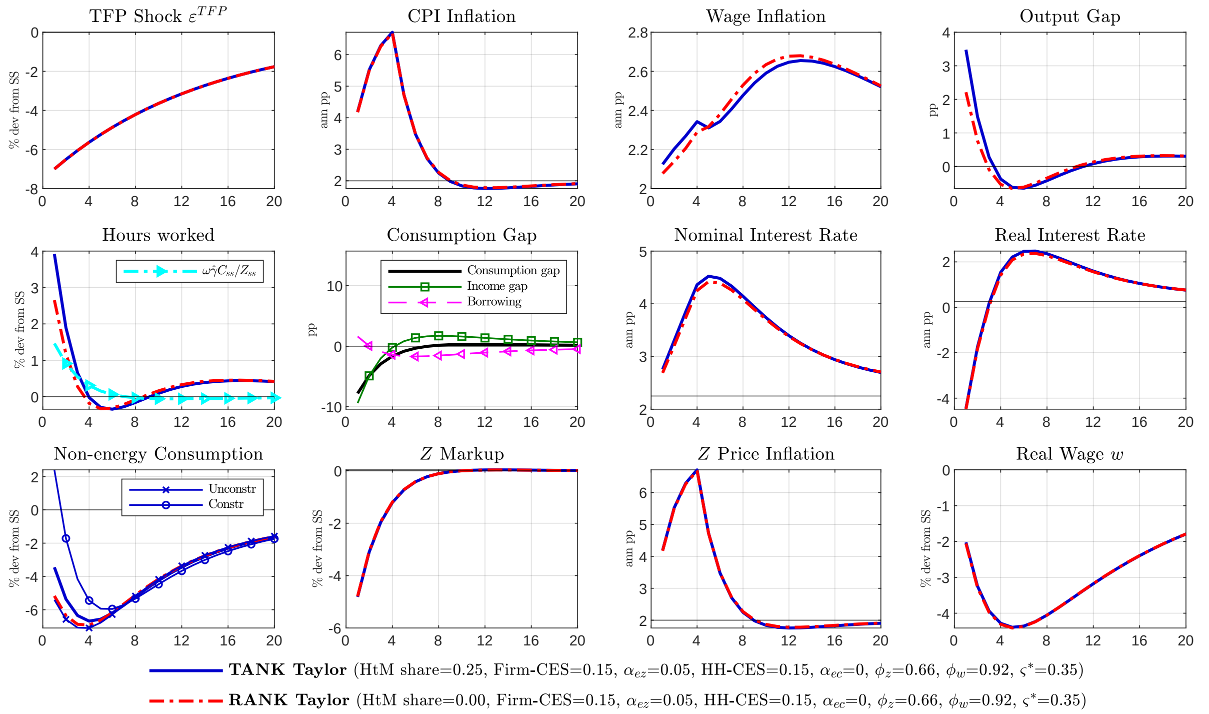
4 The Demand-side Effects of Alternate Supply Shocks

Can the economic effects of an energy price shock be appropriately proxied with a TFP shock, since both shocks constrain supply? In this section, we explore whether the demand contraction that follows a rise in energy prices is a common feature of supply disturbances.

TFP Shock IRFs Equations (2.23) to (2.25) are used to analyze the demand-side effect of a disturbance to firms' TFP. With lower productivity firms must hire more labor to produce each unit of the good. This implies lower markups for the final output firm (equation 2.24) and an increase in labour income, out of which constrained hand-to-mouth households have a high propensity to consume. It follows that the consumption gap falls and GDP increases (equation 2.20). The IRFs to an adverse TFP shock in Figure 5 illustrate this intuition. Similar to the energy price shock, the TFP shock leads to higher marginal costs, which places upward pressure on inflation. The response of the central bank to higher inflation leads to a drop in output. While both energy and TFP shocks generate similar supply-side effects, this is not the case for the demand-side effect. Lower TFP implies that *more* labor is required to produce each unit of the good, which explains the observed increase in employment. Workers' income thus increases, boosting aggregate demand. As a consequence, the TANK economy (blue lines) features a milder contraction in consumption and output relative to the RANK economy (red lines). Energy and TFP shocks therefore diverge in terms of their impact on demand. Whereas the former reduces workers' income, the latter increases it, leading to a different profile for aggregate demand.[24] We conclude that no generalization can be made about the effects of supply shocks on aggregate demand, as the nature of the shock crucially affects the way in which resources are redistributed in the economy.

[24] De Giorgi and Gambetti (2017) provide empirical evidence of pro-cyclical consumption inequality in response to a TFP shock.

FIGURE 5: Dynamic Responses to a TFP Shock: Taylor policy for TANK vs RANK

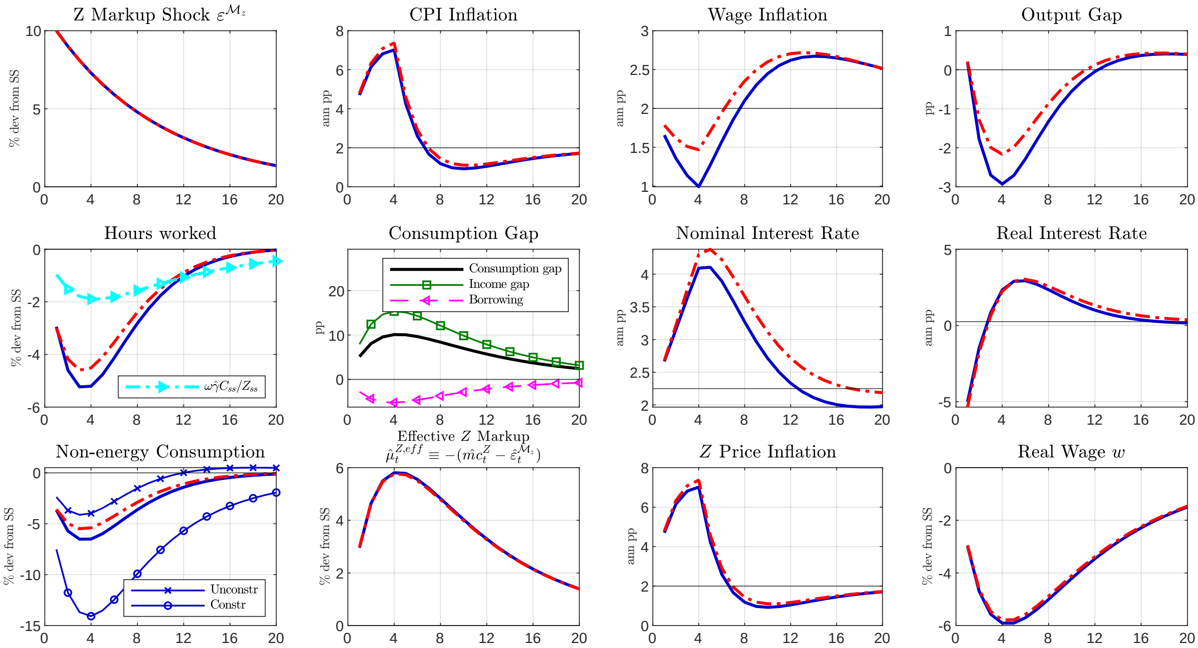


Notes: This figure shows the IRFs of key model variables to a 7% drop in TFP. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case.

Markup Shock IRFs Another supply disturbance frequently considered in the literature is a shock to the desired firm markup. In Figure 6 we show that a higher desired markup pushes inflation up, which the central bank responds to by raising the policy rate. Thus, on the supply side, the shock transmits in a similar manner to the energy and TFP disturbances. On the demand-side, higher markups imply an increase in the profit share relative to the labor share of income. The redistribution of resources against the constrained hand-to-mouth households, as captured by a rise in the consumption gap, depresses aggregate demand. This explains the deeper fall in output (proxied for by hours worked) experienced in the TANK model (blue line) relative to the RANK model (red line). Like the energy shock, a markup shock raises the consumption gap, hence depressing aggregate demand. However, in the case of a markup shock, the rise in the consumption gap is fully explained by the income gap, which goes up due to the unequal income composition between constrained hand-to-mouth households and unconstrained firm owning households. Instead, with the energy shock, the rise in the consumption gap is largely explained by an unequal access to international credit markets.

Role of Foreign Borrowing Notice that, apart from the shock to energy prices, foreign borrowing has a minor role in explaining the consumption gap after a supply disturbance. The relevance of borrowing in the case of an energy price shock is due to the impact of energy prices on the trade balance, relative to workers income (equations 2.22 to 2.25). As energy prices adversely affect the trade balance, foreign borrowing increases (equation 2.21). The increased availability of foreign borrowing is employed by the unconstrained individuals to fund their consumption, consequently increasing the consumption gap.

FIGURE 6: Dynamic Responses to a Price Markup Shock: Taylor policy for TANK vs RANK



TANK Taylor (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

RANK Taylor (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to an inflationary price markup shock. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. In the second panel of the third row we show the responses for the 'effective' final output price markup, $\hat{\mu}_t \equiv -(\hat{mc}_t^Z - \hat{\varepsilon}_t^{\mathcal{M}})$. A positive markup shock thus increases firms desired markups and hence the actual 'effective' markup.

5 Optimal Monetary Policy

To compute the Ramsey-optimal policy, we assume a utilitarian central bank that attaches equal weights to the utility of all households, maximising households' lifetime utility subject to the non-linear system of equations that describe private agents' optimality conditions. Figure 7 presents the IRFs under the Ramsey policy. We compare the optimal policy in the TANK (blue lines) versus the RANK model (red lines). The figure shows that although optimal policy leads to very similar paths for inflation and employment in the two economies, the implementation is different. In both cases, the policymaker implements contractionary policy in order to counteract the inflationary effect of the shock. However, the optimal increase in the path of the real interest rate is milder in the TANK than in the RANK. This is explained by the direct contractionary effect of higher energy prices on households' income. In the TANK, the lower income translates into lower aggregate demand, which contains the inflationary pressures of the shock. Hence, a less aggressive tightening of policy is optimal.

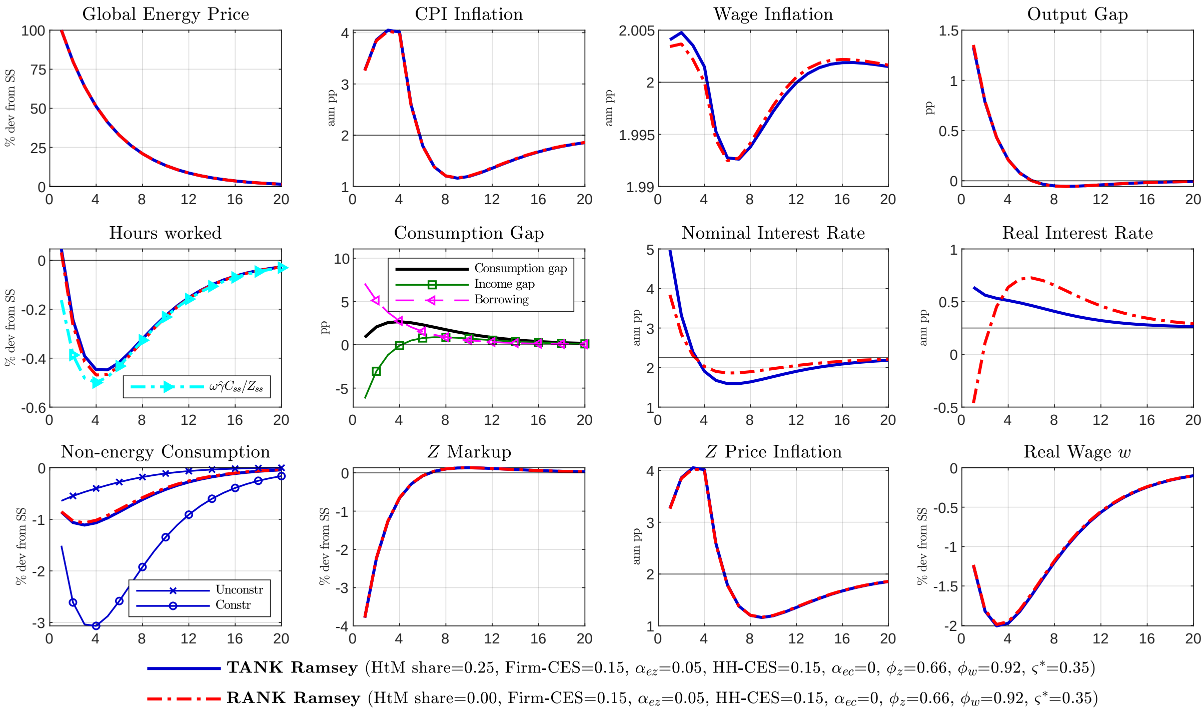
The response of the nominal interest rate on impact is stronger in the TANK than in the RANK. The nominal rate in the Ramsey-optimal TANK case is then lowered more aggressively below the steady state level of 2.25%. The policy maker in the TANK Ramsey case is thus more activist than in the RANK Ramsey case. The forceful increase in the nominal interest rate on impact counteracts the expansionary effect on current consumption of a looser policy implemented further out. The stronger aggregate demand effects in the TANK model amplifies this activist pattern.

A key difference between the Taylor-rule IRFs of Figure 2 and the Ramsey-optimal IRFs of Figure 7 is the response of wage inflation. Under the Taylor rule, wage inflation falls below the steady state of 2%[25] initially, but then increases from 1.8 % to around 2.3%. Under Ramsey-policy, wage inflation is

[25] We abstract from productivity growth in our steady state, so wages and prices both grow at 2% p.a. in the steady state.

almost perfectly stabilized. The reason for this improved stabilization of wage inflation is that under Ramsey-optimal policy, the response of the wage markup is more muted because the response of the marginal rate of substitution between consumption and leisure is closer to the real wage, compared to the Taylor-policy case. Under both RANK and TANK, Ramsey-policy achieves a substantial mitigation in the fall of hours worked and consumption. Being more accommodative under Ramsey thus cushions the fall in the MRS without affecting much the fall in the real wage w_t.[26]

FIGURE 7: Dynamic Responses to a Global Energy Price Shock: Ramsey-policy for TANK vs RANK

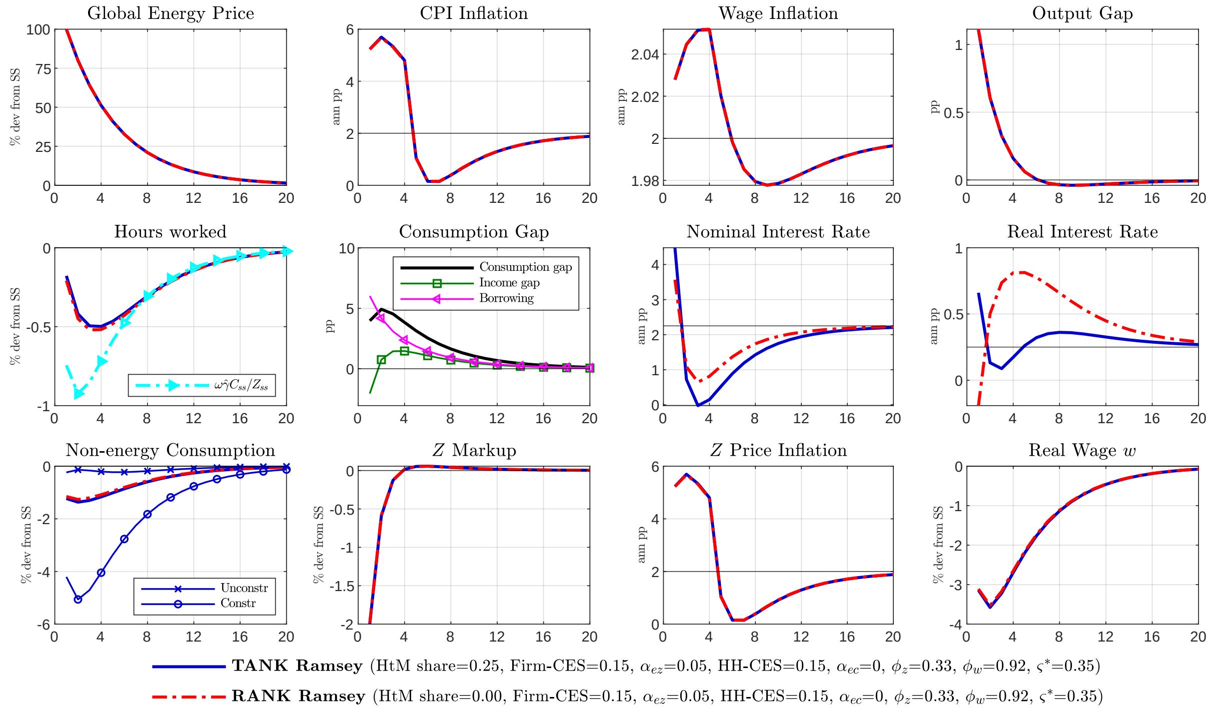


Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Full set in Figure B.3.

Role of Price Stickiness As stressed earlier, the demand effect of higher energy prices depends on the evolution of firms' markups. If inflation remains contained in spite of the costlier energy input, firms largely absorb the costs of the shock. This would be reflected in a reduction in markups. Conversely, if prices go up strongly to preserve markups, firms can pass the costs of the shock to workers, who will experience a more severe reduction in their income. The degree to which prices react to the shock thus determines who takes the hit, and hence, its impact on aggregate demand. In Figure 8 we explore a scenario where firms raise prices more aggressively in response to the costlier energy input in an attempt to preserve profits. We repeat the optimal policy exercise assuming a higher degree of price flexibility, setting the Calvo pricing parameter to $\phi_z = 0.33$. A comparison between Figure 8 and Figure 7 illustrates that when firms react to an energy price shock by raising prices strongly, constrained HtM households experience a more severe drop in their income relative to unconstrained households, as reflected by the income gap. Since the constrained households are more severely affected, there is a deeper contraction in aggregate demand. As a consequence, optimal monetary policy in the TANK model is now much looser relative to its RANK counterpart.

[26] Refer to Appendix, Section A.10 for more details.

FIGURE 8: Dynamic Responses to a Global Energy Price Shock: Ramsey-policy with Higher Price Flexibility



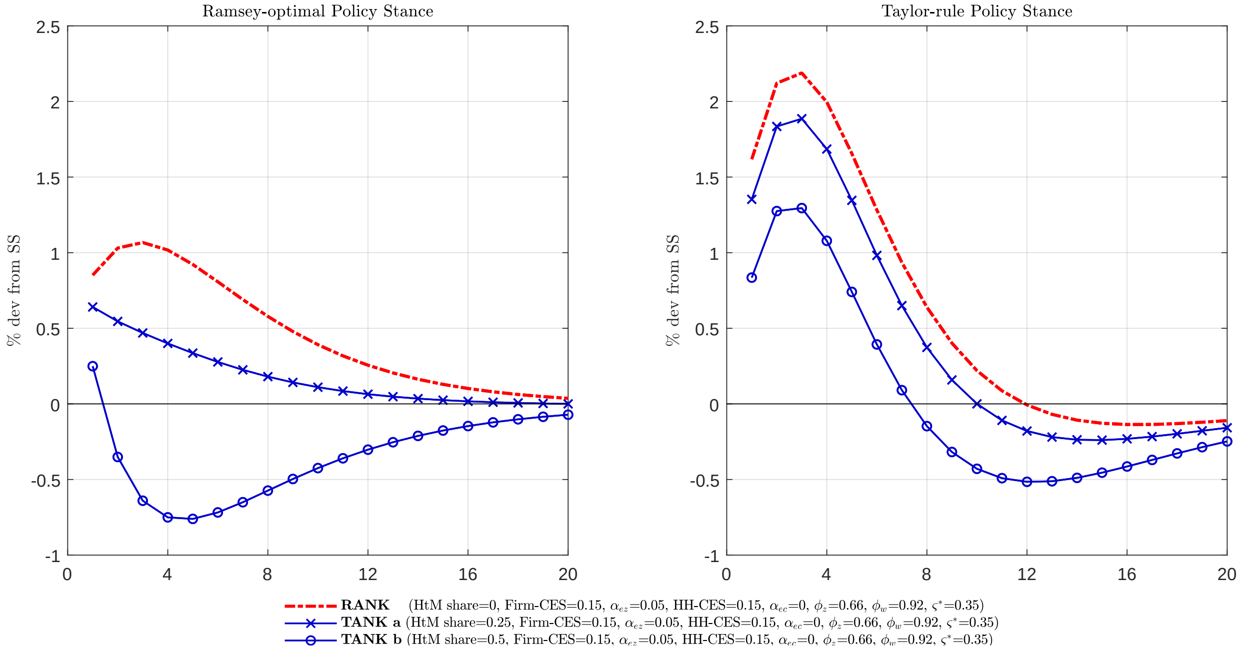
Notes: This figure shows the IRFs to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function. Full set of IRFs in Figure B.13.

Role of Credit Constraints Next, we explore whether optimal policy may actually be expansionary in response to an adverse supply shock. We can expect that as the contractionary effect of the shock on demand strengthens, it should be optimal for the policymaker to loosen policy. For this exercise, we introduce a measure for the monetary policy stance, which indicates whether policy is contractionary or expansionary. From (2.2) we know that the demand of unconstrained households, whose consumption responds to interest rates, is determined by the expected and cumulated path of the real interest rate, rather than the current spot real rate. Therefore, we define the *policy stance* as

$$\hat{st}_t \equiv \sigma^{-1}\mathbb{E}_t \sum_{k=0}^{\infty} \left(\hat{r}_t - \hat{\pi}_{t+k+1}\right). \tag{5.1}$$

Figure 9 presents the IRFs for the policy stance over an increasingly larger share of constrained agents, which allows the energy price shock to yield a correspondingly larger fall in households' consumption. The left panel in the chart depicts the case of Ramsey-optimal policy, the right panel depicts the case of a Taylor rule. In the RANK model, Ramsey-optimal monetary policy remains contractionary throughout the period of higher energy prices in order to counteract inflation. Meanwhile, in the TANK, optimal policy under a HtM weight of $\omega = 0.25$ is less contractionary. Under a higher share of constrained HtM agents, $\omega = 0.5$, the optimal policy stance can even be accomodative, as can be seen in the circled blue line in the left panel in Figure 9.

FIGURE 9: Dynamic Responses to a Global Energy Price Shock: Policy with Stronger Credit Constraints



Notes: This figure shows the IRFs of the policy stance ($C_{u,t}^{-1}$) to a 100 % increase in the foreign currency price of energy. In the panel on the left (right) the central bank implements Ramsey-optimal policy (follows a Taylor rule). Energy enters only in the firm's production function. The red lines depict the RANK case. The blue crossed (circled) line depicts the TANK case *a* (*b*) with $\omega = 0.25$ ($\omega = 0.5$). Full set in B.4.

6 Extension: Energy as a Consumption Good

In this section, we extend our model to incorporate imported energy as a component of households' consumption baskets. To be precise, assume now that unconstrained and constrained households' consumption is respectively given by the following CES aggregators

$$CES_{j,t} = \left((1-\alpha_{ec})^{\frac{1}{\psi_{ec}}} \left(C_{j,t}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{ec}^{\frac{1}{\psi_{ec}}} \left(E_{j,t}^{h}\right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}} \qquad \forall j \in \{u, c\}$$

where $C_{u,t}$ is consumption of the domestically produced good for unconstrained households, $E_{u,t}^{h}$ is energy consumption for unconstrained households, and α_{ec} denotes the share of energy in households' expenditure. The analogous variables with subscript c denote the counterparts for constrained households. Parameter ψ_{ec} is the elasticity of substitution between the domestic good and energy.

Dynamic IS Equation Analogous to the case in which energy only enters the firm's production function, we can derive a dynamic IS equation for employment $\hat{n}_t$, a proxy for 'aggregate demand', broken down by channels

$$\hat{n}_t = \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\hat{r}_{t+k} - \hat{\pi}_{t+k+1}^{CPI}\right)}_{\text{Inter-temporal substitution (-)}} \quad \underbrace{-\omega\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\substack{\text{Demand effect from}\\ \text{credit constraints (+/-)}}} \quad \underbrace{+\psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\hat{p}_t^{E} - \hat{w}_t\right)}_{\substack{\text{Intra-temporal substitution}\\ \text{in production (+)}}} \quad \underbrace{+\frac{X_{ss}}{Z_{ss}}\varsigma^*\hat{q}_t}_{\substack{\text{terms of}\\ \text{trade (+)}}}$$
$$-\hat{\varepsilon}_t^{TFP} \quad \underbrace{+\alpha_{ec}\psi_{ec}\frac{C_{ss}}{Z_{ss}}\hat{p}_t^{E}}_{\substack{\text{Intra-temporal substitution}\\ \text{in consumption (+)}}} \tag{6.1}$$

Note that CPI inflation is a function of final output price inflation $\hat{\pi}_t$ and of energy price inflation $\hat{\pi}_t^{CPI} = \hat{\pi}_t + \alpha_{ec}\Delta\hat{p}_t^E$. In section 2, in which energy only entered the firm's production function, we had $\alpha_{ec} = 0$ so that CPI inflation was equal to final output price inflation $\hat{\pi}_t^{CPI} = \hat{\pi}_t$. We can see from equation 6.1 that if energy also enters in the consumption basket, there is an additional channel through which energy affects aggregate demand: the intra-temporal substitution in consumption. Since the energy good has become more expensive in real terms, the household substitutes towards the relatively cheaper domestically produced non-energy good, which increases demand and hence employment. However, this channel is quantitatively not very relevant due to the low degree of substitutability and the low energy weight in consumption.

Moreover, since energy now directly enters the consumption basket, an increase in energy prices will directly affect CPI inflation and trigger a corresponding policy response. Energy in the consumption basket thus amplifies the role of the policy-related 'intra-temporal substitution' channel.

Next, we discuss how energy shocks affect the consumption gap $\hat{\gamma}_t = \log(\Gamma_t/\bar{\Gamma})$ in this extension.

The Consumption Gap With energy entering the consumption basket, the consumption gap (the ratio of the unconstrained household consumption bundle relative to the constrained household consumption bundle) is given by

$$\Gamma_t \equiv \frac{CES_{u,t}}{CES_{c,t}} = 1 + \underbrace{\frac{1}{(1-\omega)}\frac{(\mathcal{M}_t^Z - 1)}{\Xi_t^N}}_{\text{Income gap}} - \underbrace{\frac{tb_t}{(1-\omega)p_t^{CPI}CES_{c,t}}}_{\text{Borrowing}}. \tag{6.2}$$

With energy only in the consumption basket, the consumption gap depends positively on firms' average markups ($\mathcal{M}_t^Z$) and positively on the CPI (and hence positively on the price of energy p_t^E). We can shed further light on the dependence of the consumption gap on markups and energy prices by deriving a log-linear expression for equation (6.2)[27]

$$\begin{aligned}\Gamma\hat{\gamma}_t \quad = \quad & \underbrace{\frac{1}{1-\omega}\left(M_1\mathcal{M}\hat{\mu}_t^Z + M_2(\mathcal{M}-1)(1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)\right)}_{\text{Income gap}} + \qquad (6.3)\\ & \underbrace{\frac{B_1}{1-\omega}(1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right) + \frac{\mathcal{M}}{1-\omega}\left(B_2(\widehat{ces}_t - \hat{x}_t) + B_3\hat{\mu}_t^Z + B_4\left(1 - \frac{C}{Z}\psi_{ec}\right)\hat{p}_t^E\right)}_{\text{Borrowing}},\end{aligned}$$

where we introduced the auxiliary terms $M_1 > 1, M_2 > 0, B_1 > 0, B_2 > 0, B_3 < 0, B_4 > 0$.[28] In order to focus on the effects of energy in the consumption basket, we assume that energy does not enter the production function so that $\alpha_{ez} = 0$ and $M_2 = B_1 = 0$.

Effect of Increase in Markup An increase in the final output firm's markup increases the consumption gap ($\partial\hat{\gamma}_t/\partial\hat{\mu}_t^Z > 0$).[29] Note, however, that if energy only enters in the consumption basket then the price of energy would have no direct effect on markups ($\partial\hat{\mu}_t^Z/\partial\hat{p}_t^E = 0$). This is a key difference relative to the model with energy as a production input (equation 2.24). In that case, the higher price of energy

[27] Refer to the Appendix, Equation (A.69), for a detailed derivation.

[28] $M_1 \equiv \frac{wN+E^z}{wN} > 1$, $M_2 \equiv \frac{E^z}{wN} = M_1 - 1 > 0$ and $B_1 \equiv M_2\left(1 + \mathcal{M}\left(\frac{CES}{Z} - 1\right)\right) > 0$, $B_2 \equiv M_1\frac{CES}{Z}\frac{X}{Z} > 0$, $B_3 \equiv M_1\left(\frac{CES}{Z} - 1\right) < 0$, $B_4 \equiv M_1\frac{E^h}{Z} > 0$ Note that CES/Z = (1-X/Z)=0.75 in our calibration.

[29] Note that $\partial\hat{\gamma}_t/\partial\hat{\mu}_t^Z = \mathcal{M}M_1/(1-\omega)(CES/Z) > 0$.

gradually passed through to the price of the consumption good. Therefore, firms would partially absorb the impact of the costlier energy through a fall in markups. However, with energy in the consumption basket, energy prices instantaneously pass through to the price of the consumption good. Without a fall in markups to absorb the shock, constrained HtM households are more strongly affected by higher energy prices.

Effect of Increase in Energy Price An increase in energy prices will increase the consumption gap ($\partial\hat{\gamma}_t/\partial\hat{p}^E_t > 0$) as long as the substitutability between energy and non-energy in consumption is low enough, $\psi_{ec} < Z/C$.[30] The intuition is that the share of domestic goods in households' expenditure responds to changes in the price of energy, and the direction of this response depends on the elasticity of substitution between the domestic good and energy (ψ_{ec}).

With a low elasticity of substitution, less resources are spent on the purchase of domestic products following an increase in energy prices, hence households' income drops. While the unconstrained households are able to maintain their consumption levels by borrowing from the foreign sector, the constrained workers must cut demand. The consumption gap therefore increases and aggregate demand falls. If the elasticity of substitution is very large, the opposite is true, and higher energy prices reduce the consumption gap and boost demand. Notice that changes in the relative price of energy ($\hat{p}^E_t$) do not affect the income gap. They affect the consumption gap only because households have unequal access to credit (i.e., through the borrowing term in equation (6.2)).

Impulse Response Functions The IRFs in Figure 10 show the response to an energy price shock when energy is a component of households' consumption basket. Two additional parameters are needed relative to the baseline model calibration in Table 1: the proportion of energy in the consumption basket of households $\alpha_{ec} = 0.05$ and $\psi_{ec} = 0.15$.

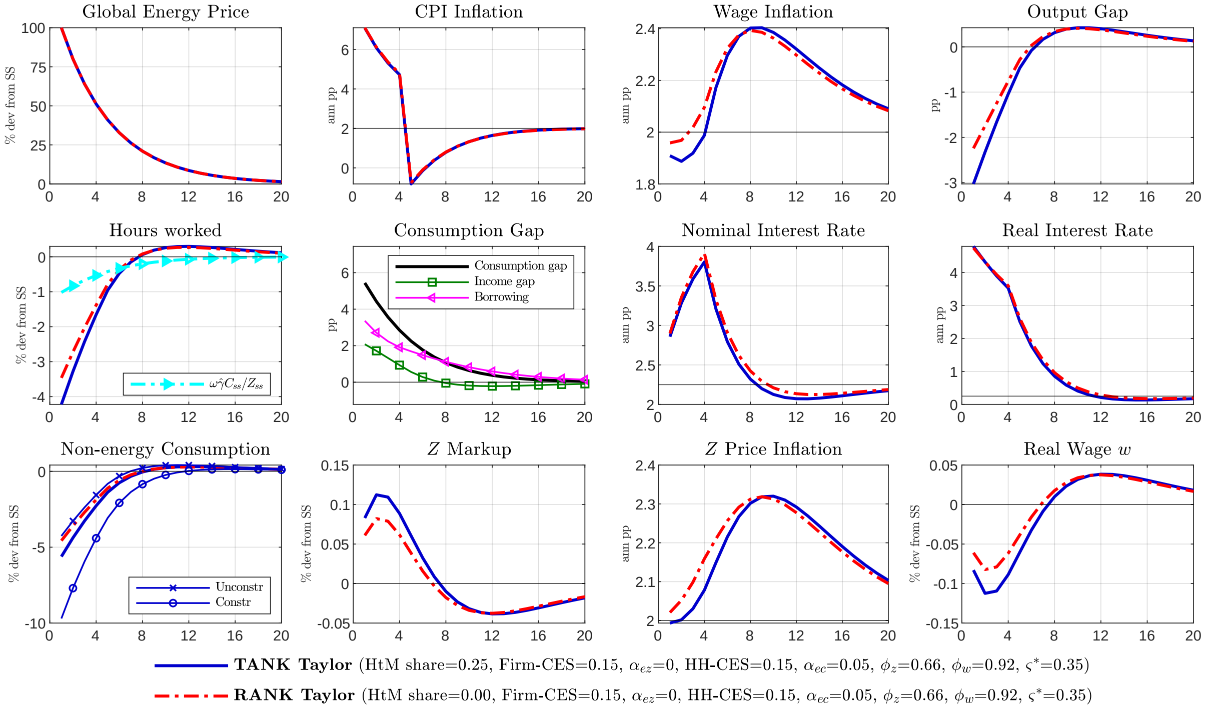
Households react to costlier energy by substituting it with the domestically produced good. However, given a low elasticity of substitution, the share of the domestically produced good in total households' expenditure decreases. Consequently, domestic households' income falls. This leads to a decline in constrained agents' consumption, resulting in a larger consumption gap and lower aggregate demand. As a result, the TANK economy experiences a more severe contraction than the RANK economy.

Compared to the scenario where energy is only a production input, there is a larger impact on inequality, as we can see by the greater increase in the consumption gap in Figure 10. This is due to the income gap, which now goes up. The different evolution of the income gap relative to the case where energy is used as an input follows from the different response of firms' markups. With energy as an input for firms, production costs would go up due to the costlier energy. Due to price rigidities, markups would fall, partially absorbing the shock. With energy entering directly in households' consumption basket, markups do not attenuate the impact of the shock on inequality. Moreover, markups now increase. This is explained by the behavior of wages, which decrease as a consequence of a weaker economy.

In summary, this exercise demonstrates that the transmission of an energy price shock is similar when energy enters firms' production function and households' consumption basket. However, the key difference lies in the impact on inequality, as the income gap increases by more when energy is a consumption good rather than a production input.

[30] The threshold $Z/C = 1/(1-0.25) = 1.333$ in our calibration.

FIGURE 10: Dynamic Responses to a Global Energy Price Shock: Energy only as a Consumption Good



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the consumption basket. Full set in Figure B.5.

7 Conclusion

We build an open economy model with household heterogeneity and complementarity between energy and labor to highlight the demand-side effects of an energy price shock. We show that an energy price shock has different effects on households, depending on their sources of income and borrowing constraints. An energy price shock leads to a fall the labor share of total factor expenditures, reducing the flow of income accruing to domestic inputs, which depresses aggregate demand. The fall in aggregate demand is stronger in the two-agent model (TANK), compared to the representative agent model (RANK). The demand-side effect of an energy price shock in our model implies that Ramsey-optimal monetary policy is less contractionary relative to a RANK model. In some cases, it may even be expansionary (i.e., when credit constraints are severe).

In our model, an energy price shock has features of an adverse productivity shock, but there are important differences. Although the supply-side effects of both shocks are the same in our model, the demand-side effect is completely different. Both an adverse productivity shock and an energy price shock lead to an increase in inflation. However, while a negative productivity shock redistributes resources towards constrained worker households, the opposite is true for an energy price shock.

A markup shock differs from an energy price shock in its effect on the consumption gap. Both shocks depress aggregate demand through a rising consumption gap, but the underlying causes for the increased gap vary. A markup shock is mainly explained by the unequal income composition between constrained households and unconstrained firm-owning households, leading to a redistribution of resources. In contrast, an energy price shock's consumption gap is largely attributed to an unequal access to international credit markets due to its direct impact on the trade balance.

We find similar results in an extension with energy in the consumption basket. Due to complementarities between energy and domestically produced goods in households' consumption baskets, higher energy prices lead to a reduction in the share of domestic goods in households' spending. As less resources are devoted to the purchase of domestically produced goods, households' income falls. While unconstrained worker households can maintain their consumption levels by borrowing from the foreign sector, constrained worker households must reduce their consumption, causing inequality to rise and aggregate demand to decline. Unlike the case of energy as a production input, markups no longer absorb the effect of the energy price shock, which exacerbates the impact of the shock on inequality.

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A Model Derivations

A.1 Households

A share $0 < \omega < 1$ of all households have access to domestic and international financial markets and are able to save and borrow in an unconstrained manner. The remaining share, $1-\omega$, are 'constrained' households. Those households directly consume their labor income. Unconstrained (constrained) household quantities are denoted with subscript u (c).

Unconstrained Households Members of unconstrained households consume, work, save, pay taxes and receive profits from firm ownership. Unconstrained household maximises their lifetime utility $\mathcal{U}_{u,s}$

$$\mathcal{U}_{u,s} = \mathbf{E}_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_{u,t} \left(C_{u,t}, E^c_{c,t}, N^h_{u,t} \right) \right\} \right], \qquad \text{where} \quad U_{u,t} = \left[\frac{(CES_{u,t})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(N^h_{u,t} \right)^{1+\varphi}}{1+\varphi} \right]$$
$$\text{and} \quad CES_{u,t} = \left((1-\alpha_{ec})^{\frac{1}{\psi_{ec}}} (C_{u,t})^{\frac{\psi_{ec}-1}{\psi_{ec}}} + (\alpha_{ec})^{\frac{1}{\psi_{ec}}} \left(E^h_{u,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}.$$

$N^h_{u,t}$ is the labour supplied by the unconstrained household, φ is the inverse Frisch elasticity of labour supply and χ is the relative weight on the dis-utility of working. The total consumption bundle consumed by the unconstrained agent, $CES_{u,t}$, is a CES composite of a domestically produced non-energy consumption good $C_{u,t}$ and of an imported energy consumption good $E^h_{u,t}$ (where the superscript h indicates *h*ousehold rather than firm demand for energy). ψ_{ec} denotes the degree of the elasticity of substitution between non-energy consumption and energy consumption, α_{ec} denotes the share of energy in consumption. Utility is maximised subject to the budget constraint

$$W^h_t N^h_{u,t} + R_{t-1} B_{u,t-1} + \bar{R}^* B^*_{u,t-1} \mathcal{E}_t + DIV_{u,t} \quad = \quad P^C_t C_{u,t} + P^E_t E^h_{u,t} + B_{u,t} + B^*_{u,t} \mathcal{E}_t + T_{u,t} + P_t \mathcal{T}_u$$

where P^C_t is the price of the final consumption bundle, P_t is the price of final output and of the domestically produced non-energy consumption good, P^E_t is the price of energy in domestic currency, paid to the domestic firm that imports energy goods from abroad (i.e. a local gas station). W^h_t denotes the nominal wage received by households, $B_{u,t}$ and $B^*_{u,t}$ denote domestic and foreign nominal debt holdings, which provide a nominal gross returns of R_t and $\bar{R}^*$ to the household. $\mathcal{E}_t$ denotes the nominal exchange rate (domestic currency relative to foreign currency), $DIV_{u,t} = DIV^F_{u,t} + DIV^L_{u,t}$ are the profits made by monopolistic firms (F) and unions (L) that are re-distributed lump-sum to unconstrained households. Total firm profits consist of final output (Z) firm profits $DIV^F_{u,t} = DIV^Z_{u,t}$, $T_{u,t} = T^F_{u,t} + T^L_{u,t}$ are a lump-sum taxes imposed on unconstrained households (to subsidize firms costs in order to get a steady state in which the distortion from monopolistic competition is eliminated). $\mathcal{T}_u$ is a steady-state transfer from unconstrained to the constrained household in order to equate their steady state level of consumption.

Total Consumption Expenditure Unconstrained households maximise their expenditure

$$\max_{C_{u,t}, E^h_{u,t}} \left\{ P^{CPI}_t CES_{u,t} - P^C_t C_{u,t} - P^E_t E^h_{u,t} \right\} \quad s.t. \quad CES_{u,t} = \left((1-\alpha_{ec})^{\frac{1}{\psi_{ec}}} C_{u,t}^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{ec}^{\frac{1}{\psi_{ec}}} \left(E^h_{u,t} \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}$$

which implies $\frac{\partial CES_{u,t}}{\partial C_{u,t}} = \frac{P^C_t}{P^{CPI}_t}$ and $\frac{\partial CES_{u,t}}{\partial E^h_{u,t}} = \frac{P^E_t}{P^{CPI}_t}$ so that the relative demand schedules are given by

$$C_{u,t} \quad = \quad \left(\frac{p^C_t}{p^{CPI}_t} \right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{u,t} \tag{A.1}$$

$$E^h_{u,t} \quad = \quad \left(\frac{p^E_t}{p^{CPI}_t} \right)^{-\psi_{ec}} (\alpha_{ec}) CES_{u,t}. \tag{A.2}$$

Optimality also implies

$$
\begin{aligned}
p_t^{CPI} CES_{u,t} &= p_t^C C_{u,t} + p_t^E E_{u,t}^h, \qquad P_t^{CPI}/P_t = p_t^{CPI}, P_t^C/P_t = p_t^C, P_t^E/P_t = p_t^E \\
\left(p_t^{CPI}\right)^{1-\psi_{ec}} &= (1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}} \\
p_t^{CPI} &= \left[(1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}} \qquad (A.3)
\end{aligned}
$$

where P_t is the domestic final output price level and P_t^{CPI} is the price of the CES bundle (the 'consumer price index').

Lagrangian Unconstrained households solve the following Lagrangian

$$
\begin{aligned}
\mathcal{L}_{u,t} = \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \Bigg\{ U_{u,t} + \Lambda_{u,t} \Bigg[& W_t^h N_{u,t}^h + R_{t-1} B_{u,t-1} + \bar{R}^* B_{u,t-1}^* \mathcal{E}_t \\
& + DIV_{u,t}^F + DIV_{u,t}^L - P_t^{CPI} CES_{u,t} - B_{u,t} - B_{u,t}^* \mathcal{E}_t - T_{u,t}^F - T_{u,t}^L - P_t \mathcal{T}_u \Bigg] \Bigg\}
\end{aligned}
$$

$\mathcal{S}^t$ and $\pi_{\mathcal{S}^t}$ denote the state of the world and the corresponding probability and $\Lambda_{u,t}$ is the Lagrange multiplier for with the unconstrained households' resource constraint.

Optimal Choice of CES_u The first-order condition for unconstrained CES-composite consumption is

$$
\Lambda_{u,t} = \frac{(CES_{u,t})^{-\sigma}}{P_t^{CPI}}, \quad \lambda_{u,t} \equiv (CES_{u,t})^{-\sigma}. \qquad (A.4)
$$

where we define the marginal utility of unconstrained CES-composite consumption as $\lambda_{u,t}$.

MRS of unconstrained HH We define the marginal rate of substitution of the unconstrained household as $MRS_{u,t} \equiv -U_{u,t}^N/\Lambda_{u,t}$ which in real terms is

$$
\begin{aligned}
mrs_{u,t} &= -\frac{U_{u,t}^N}{\lambda_{u,t}/p_t^{CPI}} \qquad (A.5) \\
U_{u,t}^N &= -\chi\left(N_{u,t}^h\right)^{\varphi} \qquad (A.6)
\end{aligned}
$$

Optimal Choice of B_u and B_u^* The domestic saving/CES-consumption Euler equation is then given by

$$
(\Lambda_{u,t}) = \beta E_t\left(\Lambda_{u,t+1} R_t\right), \quad \frac{1}{P_t^{CPI}} \lambda_{u,t} = \mathbf{E}_t\left[\beta \frac{1}{P_{t+1}^{CPI}} \lambda_{u,t+1}\right] R_t, \quad 1 = \mathbf{E}_t\left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi_{t+1}^{CPI}}\right] R_t. \qquad (A.7)
$$

$$
\text{where } \Pi_t^{CPI} \equiv \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \frac{P_t}{P_{t-1}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t \qquad (A.8)
$$

and $\Pi_t = P_t/P_{t-1}$ refers to domestic final output price inflation. The foreign saving-consumption Euler equation is as follows

$$
\begin{aligned}
\frac{\partial \mathcal{L}_{u,t}}{\partial B_{u,t}^*} &= \pi_{\mathcal{S}^t} \beta^t \left(\Lambda_{u,t}\left[-\mathcal{E}_t\right]\right) + \beta^{t+1} \sum_{\mathcal{S}^{t+1} > \mathcal{S}^t} \pi_{\mathcal{S}^{t+1}} \left(\Lambda_{u,t+1} \bar{R}^* \mathcal{E}_{t+1}\right) = 0 \\
1 &= \beta \mathbf{E}_t\left(\frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \bar{R}^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \Leftrightarrow 1 = \beta \mathbf{E}_t\left(\frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi_{ss}^*}\right)
\end{aligned}
$$

We use the definition of the real exchange rate $\mathcal{Q}_t \equiv \mathcal{E}_t P_t^* / P_t$. P_t (P_t^*) denotes the domestic (foreign) final output price level. We can derive the uncovered interest rate parity (UIP) condition

$$\mathbf{E}_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi_{t+1}^{CPI}} \left(R_t - \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi_{ss}^*} \right) \right] = 0. \tag{A.9}$$

We define the unconstrained household's stochastic discount factor as

$$\Lambda_{u,t,t+1} \equiv \mathbf{E}_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \right] \tag{A.10}$$

Unconstrained Household Budget in real terms

$$\begin{aligned} w_t^h N_{u,t}^h + \frac{R_{t-1} b_{u,t-1}}{\Pi_t} + \frac{\bar{R}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_{u,t}^F + div_{u,t}^L &= p_t^C C_{u,t} + p_t^E E_{u,t}^h + b_{u,t} + b_{u,t}^* \mathcal{Q}_t \\ &\quad + t_{u,t}^F + t_{u,t}^L + \mathcal{T}_u \end{aligned} \tag{A.11}$$

Detrending Total Profits from Firm Ownership

$$DIV_{u,t}^F = DIV_{u,t}^Z, \quad div_{u,t}^F \equiv \frac{1}{P_t} DIV_{u,t}^F, \quad div_{u,t}^F = div_{u,t}^Z. \tag{A.12}$$

Constrained Households Members of constrained households consume and work to maximise their lifetime utility $\mathcal{U}_{c,s}$

$$\mathcal{U}_{c,s} = \mathbf{E}_s \left[\sum_{t=s}^{\infty} \beta^t \left\{ U_{c,t} \left(C_{c,t}, E_{c,t}^h, N_{c,t}^h \right) \right\} \right], \quad U_{c,t} = \left[\frac{(CES_{c,t})^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(N_{c,t}^h \right)^{1+\varphi}}{1+\varphi} \right]$$

$$\text{and} \quad CES_{c,t} = \left((1-\alpha_{ec})^{\frac{1}{\psi_{ec}}} (C_{c,t})^{\frac{\psi_{ec}-1}{\psi_{ec}}} + (\alpha_{ec})^{\frac{1}{\psi_{ec}}} \left(E_{c,t}^h \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}.$$

where $CES_{c,t}$ is a CES composite of domestically produced non-energy goods $C_{u,t}$ and of imported energy goods $E_{c,t}^h$. The weight of energy in the consumption composite for the constrained household is also α_{ec}. $N_{c,t}^h$ is the constrained household's labor supply and φ is the elasticity of labor supply, χ is the relative weight on the disutility of working. Utility is maximised subject to the budget constraint

$$W_t^h N_{c,t}^h + DIV_{c,t}^L = P_t^C C_{c,t} + P_t^E E_{c,t}^h + T_{c,t}^L - P_t \mathcal{T}_c$$

where $DIV_{c,t}^L$ are the profits made by monopolistically competitive labor unions and $T_{c,t}^L$ is a transfer to the union in order to subsidize its cost.

Total Consumption Expenditure Constrained households maximise their expenditure

$$\max_{C_{c,t}, E_{c,t}^h} \left\{ P_t^{CPI} CES_{c,t} - P_t^C C_{c,t} - P_t^E E_{c,t}^h \right\} \quad s.t. \quad CES_{c,t} = \left((1-\alpha_{ec})^{\frac{1}{\psi_{ec}}} C_{c,t}^{\frac{\psi_{ec}-1}{\psi_{ec}}} + \alpha_{ec}^{\frac{1}{\psi_{ec}}} \left(E_{c,t}^h \right)^{\frac{\psi_{ec}-1}{\psi_{ec}}} \right)^{\frac{\psi_{ec}}{\psi_{ec}-1}}$$

so that the relative demand schedules are given by

$$C_{c,t} = \left(p_t^C / p_t^{CPI} \right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{c,t} \tag{A.13}$$

$$E_{c,t}^h = \left(p_t^E / p_t^{CPI} \right)^{-\psi_{ec}} (\alpha_{ec}) CES_{c,t}. \tag{A.14}$$

Optimality also implies

$$P_t^{CPI} CES_{c,t} = P_t^C C_{c,t} + P_t^E E_{c,t}^h, \qquad p_t^{CPI} = \left[(1-\alpha_{ec}) \left(p_t^C \right)^{1-\psi_{ec}} + \alpha_{ec} \left(p_t^E \right)^{1-\psi_{ec}} \right]^{\frac{1}{1-\psi_{ec}}}. \tag{A.15}$$

Lagrangian Each constrained household solves the following Lagrangian in any arbitrary period t

$$\mathcal{L}_{c,t} = \sum_{\mathcal{S}^t} \pi_{\mathcal{S}^t} \sum_{t=0}^{\infty} \beta^t \left\{ U_{c,t} + \Lambda_{c,t} \left[W_t^h N_{c,t}^h + DIV_{c,t}^L - P_t^{CPI} CES_{c,t} - T_{c,t}^L + P_t \mathcal{T}_c \right] \right\}$$

where $\Lambda_{c,t}$ is the constrained household Lagrange multiplier associated with the resource constraint.

Optimal Choice of CES The first-order condition for constrained CES-composite consumption is

$$\Lambda_{c,t} \quad = \quad \frac{U_{c,t}^{CES}}{P_t^{CPI}}, \quad U_{c,t}^{CES} = (CES_{c,t})^{-\sigma}, \quad \lambda_{c,t} \equiv (CES_{c,t})^{-\sigma} \tag{A.16}$$

where we define the marginal utility of constrained household consumption as $\lambda_{c,t}$.

MRS of constrained HH The real marginal rate of substitution of the constrained household is

$$mrs_{c,t} \quad = \quad -\frac{U_{c,t}^N}{(CES_{c,t}^{-\sigma})/p_t^{CPI}} \tag{A.17}$$

$$U_{c,t}^N \quad = \quad -\chi \left(N_{c,t}^h \right)^{\varphi} \tag{A.18}$$

Real constrained household budget

$$p_t^C C_{c,t} + p_t^E E_{c,t}^h \quad = \quad w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L + \mathcal{T}_c \tag{A.19}$$

We use

$$t_t^F \quad = \quad t_t^Z \tag{A.20}$$

$$t_t^Z \quad = \quad (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z) \tag{A.21}$$

$$t_t^L \quad = \quad (1-\tau_t^W) w_t^h N_t^h \tag{A.22}$$

Aggregation and Market Clearing

$$C_t \quad = \quad \omega C_{c,t} + (1-\omega) C_{u,t} \tag{A.23}$$

$$E_t^h \quad = \quad \omega E_{c,t}^h + (1-\omega) E_{u,t}^h \tag{A.24}$$

$$N_t^h \quad = \quad \omega N_{c,t}^h + (1-\omega) N_{u,t}^h \tag{A.25}$$

$$\Lambda_{t,t+1} \quad = \quad (1-\omega) \Lambda_{u,t,t+1} \tag{A.26}$$

Moreover, we have $b_t = (1-\omega) b_{u,t}$, $div_t^F = (1-\omega) div_{u,t}^F$, $div_t^L = \omega div_{c,t}^L + (1-\omega) div_{u,t}^L$, $b_t^* = (1-\omega) b_{u,t}^*$, $t_t^F = (1-\omega) t_{u,t}^F$, $t_t^L = \omega t_{c,t}^L + (1-\omega) t_{u,t}^L$.

Domestic Bond Market Clearing We assume that domestic bonds are in zero net supply

$$b_t = 0. \tag{A.27}$$

Firm and Union Profits net of monopolistic competition correction subsidy

$$
\begin{aligned}
div_t^L - t_t^L &= w_t N_t - \tau_t^W w_t^H N_t^h - w_t^H N_t^h + \tau_t^W w_t^H N_t^h = w_t N_t - w_t^H N_t^h, \quad t_t^L = (1-\tau_t^L) w_t^h N_t^h \\
div_t^Z - t_t^Z &= Z_t - \tau_t^Z \left(w_t N_t + p_t^E E_t^z \right) - t_t^Z = Z_t - \left(w_t N_t + p_t^E E_t^z \right)
\end{aligned}
$$

Combine Firm Profits net of monopolistic competion correction subsidy

$$
\begin{aligned}
div_t^F - t_t^F &= Z_t - \left(w_t N_t + p_t^E E_t^z \right) + p_t^E E_t - p_t^E E_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t \\
div_t^F - t_t^F &= Z_t - w_t N_t - p_t^E E_t^z + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t
\end{aligned}
$$

Goods Market Clearing - Combine Household Budgets Recall the real-term household budgets

$$
\begin{aligned}
p_t^C C_{u,t} &= w_t^h N_{u,t}^h + \frac{\bar{R}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_{u,t}^F + div_{u,t}^L - t_{u,t}^F - t_{u,t}^L - b_{u,t}^* \mathcal{Q}_t - \mathcal{T}_u - p_t^E E_{u,t}^h \\
p_t^C C_{c,t} &= w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L + \mathcal{T}_c - p_t^E E_{c,t}^h
\end{aligned}
$$

and re-arrange for consumption, pre-multiplied with their household-type share ω to get

$$
\begin{aligned}
p_t^C C_t &= (1-\omega) \left(w_t^h N_{u,t}^h + \frac{\bar{R}^* b_{u,t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + div_{u,t}^F - t_{u,t}^F + div_{u,t}^L - t_{u,t}^L - b_{u,t}^* \mathcal{Q}_t - \mathcal{T}_u - p_t^E E_{u,t}^h \right) \\
&\quad + \omega \left(w_t^h N_{c,t}^h + div_{c,t}^L - t_{c,t}^L + \mathcal{T}_c - p_t^E E_{c,t}^h \right), \quad (1-\omega)\mathcal{T}_u = \omega \mathcal{T}_c
\end{aligned}
$$

which can be simplified to

$$
\begin{aligned}
p_t^C C_t &= \underbrace{w_t^h N_t^h + div_t^L - t_t^L}_{=w_t N_t^h} + \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + \underbrace{div_t^F - t_t^F}_{Z_t - w_t N_t^h - p_t^E E_t^z + p_t^{EXP} \mathcal{Q}_t X_t - p_t^X X_t} \\
&\quad - b_t^* \mathcal{Q}_t - p_t^E E_t^h \\
p_t^C C_t + p_t^X X_t &= \underbrace{\left(-b_t^* \mathcal{Q}_t + \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} \right)}_{-tb_t} + Z_t + p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h) \\
p_t^C C_t + \underbrace{p_t^X X_t - \overbrace{p_t^{EXP} \mathcal{Q}_t}^{p_t^X} X_t}_{=0} + tb_t &= Z_t - p_t^E (E_t^z + E_t^h) \equiv \frac{INC_t}{P_t}
\end{aligned}
$$

and

$$
p_t^C C_t + p_t^X X_t = \left(-b_t^* \mathcal{Q}_t + \frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} \right) + Z_t + \underbrace{p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h)}_{tb_t}
$$

and finally

$$
p_t^C C_t + p_t^X X_t = Z_t \tag{A.28}
$$

and the real trade balance tb_t is defined as

$$
tb_t = p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h) = -\frac{\bar{R}^* b_{t-1}^* \mathcal{Q}_t}{\Pi_{ss}^*} + b_t^* \mathcal{Q}_t \tag{A.29}
$$

Consumption Gap Definition We define the consumption gap as the ratio between unconstrained and constrained consumption

$$\Gamma_t \equiv \frac{CES_{u,t}}{CES_{c,t}}, \quad \text{if } \alpha_{ec} = 0 \quad \Gamma_t = \frac{C_{u,t}}{C_{c,t}}. \tag{A.30}$$

Income Gap Definition We define the 'income' of the unconstrained and constrained households as

$$inc_{u,t} \equiv w_t N^h_{u,t} - \mathcal{T}_u + div^F_{u,t} - t^F_{u,t} = \underbrace{p^C_t C_{u,t} + p^E_t E^h_{u,t}}_{p^{CPI}_t CES_{u,t}} + \underbrace{\frac{\bar{R}^* b^*_{u,t-1} \mathcal{Q}_t}{\Pi^*_{ss}} - b^*_{u,t} \mathcal{Q}_t}_{= -\frac{1}{1-\omega}\left(p^E_t (E^z_t + E^h_t) - p^X_t X_t\right)} \tag{A.31}$$

$$inc_{c,t} \equiv w_t N^h_{c,t} + \mathcal{T}_c = \underbrace{p^C_t C_{c,t} + p^E_t E^h_{c,t}}_{p^{CPI}_t CES_{u,t}}. \tag{A.32}$$

The 'income gap' is the ratio between unconstrained and constrained income

$$\Gamma^{inc}_t = \frac{inc_{u,t}}{inc_{c,t}} = \frac{\overbrace{p^C_t C_{u,t} + p^E_t E^h_{u,t}}^{p^{CPI}_t CES_{u,t}} + tb_t/(1-\omega)}{\underbrace{p^C_t C_{c,t} + p^E_t E^h_{c,t}}_{p^{CPI}_t CES_{c,t}}} = \Gamma_t + \frac{tb_t/(1-\omega)}{p^{CPI}_t CES_{c,t}}. \tag{A.33}$$

It can be shown that

$$\Gamma_t = \underbrace{\Gamma^{inc}_t}_{\text{income gap component}} + \underbrace{\frac{1}{(1-\omega) p^{CPI}_t CES_{c,t}} \left(p^E_t (E^z_t + E^h_t) - p^X_t X_t\right)}_{\text{borrowing}}.$$

In the case in which households don't consume energy, $\alpha_{ec} = 0$ we would get

$$\Gamma_t = \Gamma^{inc}_t - \frac{1}{(1-\omega) C_{c,t}} \left(p^X_t X_t - p^E_t (E^z_t)\right).$$

Note that we can express the income gap as a function of the firm's markup and the labour share of total factor expenditure

$$\Gamma^{inc}_t = \frac{inc_{u,t}}{inc_{c,t}} = \frac{w_t N^h_{u,t} - \mathcal{T}_u + (div^F_t - t^F_t)/(1-\omega)}{w_t N^h_{c,t} + \mathcal{T}_c}, \quad \mathcal{T}_u = \frac{\omega}{(1-\omega)} \mathcal{T}_c$$

so that

$$\Gamma^{inc}_t = \frac{\frac{1}{1-\omega}\left((1-\omega) w_t N^h_{u,t} - \omega \mathcal{T}_c + (div^F_t - t^F_t)\right)}{w_t N^h_{c,t} + \mathcal{T}_c} = \frac{\left((1-\omega) w_t N^h_{u,t} - \omega \mathcal{T}_c + \mathcal{T}_c - \mathcal{T}_c + (div^F_t - t^F_t)\right)}{(1-\omega)(w_t N^h_{c,t} + \mathcal{T}_c)}$$

$$\Gamma^{inc}_t = \frac{inc_{u,t}}{inc_{c,t}} = \underbrace{\frac{(1-\omega)(w_t N^h_{u,t} + \mathcal{T}_c)}{(1-\omega)(w_t N^h_{c,t} + \mathcal{T}_c)}}_{=1, \text{ since } N^h_{u,t} = N^h_{c,t}} + \frac{(div^F_t - t^F_t - \mathcal{T}_c)}{(1-\omega)(w_t N^h_{c,t} + \mathcal{T}_c)}$$

$$\Gamma^{inc}_t = \frac{inc_{u,t}}{inc_{c,t}} = 1 + \frac{(div^F_t - t^F_t - \mathcal{T}_c)}{(1-\omega)(w_t N^h_{c,t} + \mathcal{T}_c)} = 1 + \frac{(Z_t - (w_t N_t + p^E_t E^z_t) - \mathcal{T}_c)}{(1-\omega)(w_t N^h_{c,t} + \mathcal{T}_c)}$$

Note that

$$\frac{P_t}{\mathcal{M}_t} Z_t = MC_t^Z Z_t = W_t N_t + P_t^E E_t^z \quad \Leftrightarrow \quad Z_t = \mathcal{M}_t \left(w_t N_t + p_t^E E_t^z \right)$$

so that

$$\begin{aligned}
\Gamma_t^{inc} &= \frac{inc_{u,t}}{inc_{c,t}} = 1 + \frac{1}{(1-\omega)} \frac{(\mathcal{M}_t - 1)\left(w_t N_t + p_t^E E_t^z\right) - \mathcal{T}_c}{(w_t N_{c,t}^h + \mathcal{T}_c)}, \quad \text{using } N_{c,t} = N_{u,t} = N_t \\
\Gamma_t^{inc} &= \frac{inc_{u,t}}{inc_{c,t}} = 1 + \frac{1}{(1-\omega)} \frac{(\mathcal{M}_t - 1)}{\Xi_t^N} - \frac{\mathcal{T}_c}{w_t N_{c,t}^h + \mathcal{T}_c}, \quad \frac{\partial \Gamma_t^{inc}}{\partial \mathcal{M}_t} > 0, \quad \frac{\partial \Gamma_t^{inc}}{\partial \Xi_t^N} < 0 \\
\Xi_t^N &\equiv \frac{w_t N_{c,t}^h + \mathcal{T}_c}{w_t N_{c,t}^h + p_t^E E_t^z}, \quad \frac{\partial \Xi_t^N}{\partial p_t^E} < 0
\end{aligned}$$

In the text above, we assume that there are no steady state transfers, $\mathcal{T}_c = \mathcal{T}_u = 0$. This assumption does not affect the first-order dynamics of the model. In the numerical simulations that underpin the IRFs, we have $\mathcal{T}_u = \omega/(1-\omega)\mathcal{T}_c$.

Decomposing Borrowing

$$\begin{aligned}
borrowing_t &= \frac{1}{(1-\omega) p_t^{CPI} CES_{c,t}} \left(p_t^E (E_t^z + E_t^h) - p_t^X X_t \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{p_t^E E_t^z + w_t N_{c,t}^h + \mathcal{T}_c - (w_t N_{c,t}^h + \mathcal{T}_c)}{w_t N_{c,t}^h + \mathcal{T}_c} + \frac{p_t^E E_t^h}{p_t^{CPI} CES_{c,t}} - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{p_t^E E_t^h + p_t^C C_t - p_t^C C_t}{p_t^{CPI} CES_{c,t}} - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{p_t^E E_t^h + p_t^C C_t}{p_t^{CES} CES_{c,t}} - \frac{p_t^C C_t}{p_t^{CES} CES_{c,t}} - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{p_t^C C_t}{p_t^{CES} CES_{c,t}} \frac{p_t^E E_t^h + p_t^C C_t}{p_t^C C_t} - \frac{p_t^C C_t}{p_t^{CES} CES_{c,t}} - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{Z_t - p_t^X X_t}{w_t N_{c,t}^h + \mathcal{T}_c} \left(\frac{1}{\Xi_t^C} - 1 \right) - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{Z_t}{w_t N_{c,t}^h + \mathcal{T}_c} \left(\frac{1}{\Xi_t^C} - 1 \right) - \frac{p_t^X X_t}{inc_{c,t}} \left(\frac{1}{\Xi_t^C} - 1 \right) - \frac{p_t^X X_t}{inc_{c,t}} \right) \\
borrowing_t &= \frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{\mathcal{M}_t}{\Xi_t^N} \left(\frac{1}{\Xi_t^C} - 1 \right) - \frac{p_t^X X_t}{inc_{c,t}} \frac{1}{\Xi_t^C} \right), \Xi_t^C \equiv \frac{p_t^C C_t}{p_t^E E_t^h + p_t^C C_t}, \text{ if } \alpha_{ec} = 0 \text{ then } \Xi_t^C = 1
\end{aligned}$$

So that the consumption gap can be expressed as

$$\Gamma_t = \underbrace{1 + \frac{1}{(1-\omega)} \frac{(\mathcal{M}_t - 1)}{\Xi_t^N} - \frac{\mathcal{T}_c}{w_t N_{c,t}^h + \mathcal{T}_c}}_{\text{Income gap}} + \underbrace{\frac{1}{1-\omega} \left(\frac{1}{\Xi_t^N} - 1 + \frac{\mathcal{M}_t}{\Xi_t^N} \left(\frac{1}{\Xi_t^C} - 1 \right) - \frac{p_t^X X_t}{inc_{c,t}} \frac{1}{\Xi_t^C} \right)}_{\text{Borrowing}}.$$

Note that

$$\Xi_t^N \equiv \frac{w_t N_{c,t}^h + \mathcal{T}_c}{w_t N_{c,t}^h + p_t^E E_t^z}, \quad \text{if } \alpha_{ez} = 0 \text{ and } \mathcal{T}_c \text{ then } \Xi_t^N = 1.$$

A.2 Labor Packers and Unions

We follow Schmitt-Grohe and Uribe (2006) and introduce wage stickiness into the model via two types of agents: (i) perfectly competitive labor packers and (ii) monopolistically competitive unions. After households have chosen how much labor to supply in a given period, $N^h_{k,t}(j)$, $k \in \{u, c\}$, this labor is supplied to a union, in return for a nominal wage W^h_t. The union unpacks the homogenous labor supplied by households and differentiates it into different varieties $N_t(j)$, $j \in [0, 1]$ and sells these units of labor varieties at wage $W_t(j)$. Due to imperfect substitutability the union can act as a monopolist.

Labor Packers Varieties $N_t(j)$ are assembled by labor packers according to a CES production function. $N_t(j)$ denotes the demand for a specific labor variety j and N_t denotes aggregate labor demand. ϵ_w is the elasticity of substitution between labor varieties and thus $\mathcal{M}_w = \epsilon_w/(\epsilon_w - 1)$ is the corresponding gross wage markup of monopolistically competitive unions. After the packers have assembled the labor bundle they sell it to firms at wage W_t who then use it in the production process. The packers' production function, and the implied demand schedule associated with the cost minimisation are

$$N_t = \left[\int_0^1 (N_t(j))^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{\frac{\mathcal{M}_w}{1-\mathcal{M}_w}} N_t, \quad W_t \equiv \left(\int_0^1 (W_t(j))^{\frac{1}{1-\mathcal{M}_w}} dj\right)^{1-\mathcal{M}_w}$$

where W_t is the aggregate wage index. Optimal packer behaviour implies that $W_t N_t = \int_0^1 W_t(j) N_t(j) dj$.

Labor Unions Each individual labor union who sells its imperfectly substitutable labor variety $N^h_t(j)$ to the packer is subject to *nominal wage rigidities*. The probability that the union cannot reset its wage is ϕ_w. It is convenient to split the problem of a monopolistically competitive labor union into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal wage setting problem.

Cost Minimisation Problem A union will choose to minimise its costs $\tau^W W^h_t N^h_t(j)$ subject to meeting the packer's labor demand. The Lagrange multiplier $MC^W_t(j)$ is the union's (nominal) shadow cost of providing one more unit of labor, i.e. the nominal marginal cost and τ^W is a subsidy to marginal costs that eliminates the steady state distortion associated with monopolistic competition. Note that the Lagrange multiplier of an individual union j does not depend on its own quantities of inputs demanded, so that all unions have the same marginal costs $MC^W_t(j) = MC^W_t$. The wage paid to households[31], W^h_t corresponds to the marginal rate of substitution so that $MC^W_t = \tau^W W^h_t = \tau^W MRS_t$. Recall that we use lower cases to denotes real (final output price level) terms $w^h_t \equiv W^h_t / P_t$ so that

$$mc^W_t = \tau^W w^h_t \tag{A.34}$$
$$w^h_t = mrs_t, \quad \text{where} \quad mrs_t = \omega mrs_{c,t} + (1-\omega) mrs_{u,t} \tag{A.35}$$

Following Galí et al. (2007b) we assume that the union takes into account the fact that firms allocate labor demand uniformly across different workers of type j, independently of their household type $\{u, c\}$

$$N^h_{u,t} = N^h_{c,t}. \tag{A.36}$$

Wage Setting The objective of each union j is to maximise its nominal profits $DIV^L_t(j)$

$$DIV^L_t(j) = W_t(j) N^h_t(j) - \left\{\tau^W \left(W^h_t N^h_t(j)\right)\right\}, \quad div^L_t = \left(w_t - mc^W_t\right) N^h_t \tag{A.37}$$

With probability ϕ_w a union is stuck with its previous-period wage indexed to a composite

$$W_t(j) = \begin{cases} W^\#_t(j) & \textit{with probability: } 1 - \phi_w \\ W_{t-1}(j)\left(\left(\Pi^W_{ss}\right)^{1-\xi_w}\left(\Pi^W_{t-1}\right)^{\xi_w}\right) & \textit{with probability: } \phi_w \end{cases}$$

[31] We assume that both unconstrained and constrained households receive the same wage.

where $\xi_w = 0$ is the weight attached to the previous period wage inflation. Consider a union who can reset its wage in the current period $W_t(j) = W_t^{\#}(j)$ and who is then stuck with its wage until future period $t+s$. The wage in this case would be

$$W_{t+s}(j) \quad = \quad W_t^{\#}(j)\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\prod_{g=0}^{s-1}\left(\left(\Pi_{t+g}^W\right)^{\xi_w}\right)\right) = W_t^{\#}(j)\left[\left(\Pi_{ss}^W\right)^{s(1-\xi_w)}\left(\frac{W_{t+s-1}}{W_{t-1}}\right)^{\xi_w}\right]$$

Subject to the above derived demand constraint and assuming that a union j always meets the demand for its labor at the current wage labor unions solve the following optimisation problem

$$\max_{W_t^{\#}(j)} E_t \sum_{s=0}^{\infty}(\phi_w)^s \Lambda_{u,t,t+s} P_{t+s}\left[\left(W_t^{\#}(j)/P_{t+s} - mc_{t+s}^W\right) N_{t+s}^h(j)\right] \quad \text{s.t. } N_{t+s}^h(j) = \left(\frac{W_t^{\#}(j)}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} N_{t+s}.$$

$$\max_{W_t^{\#}(j)} E_t \sum_{s=0}^{\infty}(\phi_w)^s \Lambda_{u,t,t+s} P_{t+s}\left[\left(\frac{W_t^{\#}(j)}{P_{t+s}} - mc_{t+s}^W\right)\left(\frac{W_t^{\#}(j)}{W_{t+s}}\right)^{-\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} N_{t+s}\right].$$

Taking the derivative with respect to $W_t^{\#}(j)$ delivers the familiar wage inflation schedule

$$\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w \quad = \quad w_t^{\#} = \frac{W_t^{\#}}{W_t} = \left(\frac{1-\phi_w(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_w}\right)^{1-\mathcal{M}_w} \tag{A.38}$$

$$f_t^{W,1} \quad = \quad N_t\frac{mc_t^W}{w_t} + \phi_w\beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}} f_{t+1}^{W,1}\right] \tag{A.39}$$

$$f_t^{W,2} \quad = \quad N_t + \phi_w\beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}^W}{\Pi_{ss}^W}\right)^{\frac{1}{\mathcal{M}_w-1}} f_{t+1}^{W,2}\right] \tag{A.40}$$

$$\zeta_t^W \quad = \quad \Pi_t^W/\Pi_{ss}^W \tag{A.41}$$

$$w_t \quad = \quad \Pi_t^W/\Pi_t w_{t-1} \tag{A.42}$$

$$\mathcal{D}_t^W \quad = \quad (1-\phi_w)\left(\frac{1-\phi_w\left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_w}\right)^{\mathcal{M}_w} + \phi_w\left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w-1}}\mathcal{D}_{t-1}^W \tag{A.43}$$

Wage dispersion is given by $\mathcal{D}^W$. Aggregate hours worked in the economy is given by $N_t^h = N_t\mathcal{D}_t^W$.

A.3 Firms

There are three domestic firm sectors in our model: (i) final output good producers, (ii) import good producers and (iii) export good producers. Final output firms are characterised by monopolistic competition and nominal rigidities.

Final Output Goods Sector Final output goods production involves two types of agents: (i) perfectly competitive final output packers and (ii) monopolistically competitive final output producers.

Final Output Good packers Final output packers demand and aggregate infinitely many varieties of final output goods $Z_t(i)$, $i \in [0,1]$ into a final output good Z_t. $Z_t(i)$ denotes the demand for a specific variety i of the final output good and Z_t denotes the aggregate demand of the final output good. ϵ_z is the elasticity of substitution and $\mathcal{M}_z = \epsilon_z/(\epsilon_z - 1)$ is the corresponding gross markup of monopolistically competitive final output good producers. Final output packers purchase a single variety at given prices $P_t(i)$ and sell the final output good Z_t at price P_t to a sectoral retailer who transforms the final output good into consumption and export goods. The packers' CES production function, and the implied demand schedule associated with the cost minimisation are

$$Z_t = \left[\int_0^1 (Z_t(i))^{1-\frac{1}{\epsilon_z}}\, di \right]^{\frac{\epsilon_z}{\epsilon_z - 1}}, \quad Z_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{\mathcal{M}_z}{1-\mathcal{M}_z}} Z_t, \quad P_t \equiv \left(\int_0^1 (P_t(i))^{\frac{1}{1-\mathcal{M}_z}}\, di \right)^{1-\mathcal{M}_z}$$

where P_t is the price index and optimal behaviour implies $P_t Z_t = \int_0^1 P_t(i) Z_t(i) di$.

Final Output Good Producers Each variety $Z_t(i)$ that the final output good packer demands and assembles is produced and supplied by a single *monopolistically competitive* final output producer $i \in [0,1]$ according to the final output CES production function

$$Z_t(i) \quad = \quad \varepsilon_t^{TFP} \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} (N_t(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_t^z(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} \tag{A.44}$$

The production inputs demanded by a specific firm i are labor $N_t(i)$ and imported energy goods $E_t^z(i)$. α_{ez} denotes the share of energy in production and ψ_{ez} denotes the elasticity of substitution between labor and the import good. Both, labour is provided by monopolistically competitive unions. Moreover, firm i purchases energy imports E_t^z from the importer. Each individual final output producer is subject to *nominal rigidities*. The probability that they cannot reset their price is ϕ_z. We split the firms problem into two steps: (i) the intra-temporal cost minimisation problem and (ii) the inter-temporal price setting problem.

Cost Minimisation Problem A final output firm chooses its inputs to minimise its costs

$$\mathcal{L}_t^Z = -\tau_t^Z \left(W_t N_t(i) + P_t^E E_t^z(i) \right) + MC_t^Z(i) \left(Z_t(i) - \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z - 1}} Z_t \right)$$

where $MC_t^Z(i)$ is the (nominal) shadow cost of producing one more unit of final output, e.g. the nominal marginal cost and $\tau_t^Z = \tau^Z \varepsilon_t^{\mathcal{M}_z}$ is a shock to final output marginal costs that is isomorphic to a price markup shock process. The optimality conditions are given by

$$w_t \quad = \quad (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{N_t(i)} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{A.45}$$

$$p_t^E \quad = \quad (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t(i)}{E_t^z(i)} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{A.46}$$

Combine the first order conditions and rearrange to obtain the optimal trade-off between production factors as a function of their relative price,

$$\frac{W_t}{P_t^E} \quad = \quad \left(\frac{1-\alpha_{ez}}{\alpha_{ez}} \right)^{\frac{1}{\psi_{ez}}} \left(\frac{N_t(i)}{E_t^z(i)} \right)^{-\frac{1}{\psi_{ez}}}, \qquad \frac{N_t(i)}{E_t^z(i)} = \frac{1-\alpha_{ez}}{\alpha_{ez}} \left(\frac{W_t}{P_t^E} \right)^{-\psi_{ez}}.$$

Factor Demand Schedules Combine the optimality condition with the production function

$$Z_t(i) \quad = \quad \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\frac{1-\alpha_{ez}}{\alpha_{ez}} \left(\frac{W_t}{P_t^E} \right)^{-\psi_{ez}} E_t^z(i) \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + \alpha_{ez}^{\frac{1}{\psi_{ez}}} (E_t^z(i))^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}$$

$$Z_t(i) \quad = \quad \alpha_{ez}^{-1} \left(\frac{(1-\alpha_{ez}) W_t^{1-\psi_{ez}} + \alpha_{ez} (P_t^E)^{1-\psi_{ez}}}{(P_t^E)^{1-\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} E_t^z(i)$$

Rearrange to obtain the demand function for $E_t^z(i)$ and for $N_t(i)$

$$
\begin{aligned}
E_t^z(i) &= \alpha_{ez}\left(\frac{P_t^E}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}}+\alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}}\right)^{-\psi_{ez}} Z_t(i).\\
N_t(i) &= (1-\alpha_{ez})\left(\frac{W_t}{\left((1-\alpha_{ez})W_t^{1-\psi_{ez}}+\alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}}\right)^{-\psi_{ez}} Z_t(i).
\end{aligned}
$$

Final Output Marginal Cost To obtain the marginal cost, raise the first order condition with respect to $N_t(i)$ to the power $1-\psi_{ez}$ and multiply by $1-\alpha_{ez}$, re-arrange and obtain

$$
\begin{aligned}
(1-\alpha_{ez})W_t^{1-\psi_{ez}} &= (1-\alpha_{ez})^{\frac{1-\psi_{ez}}{\psi_{ez}}+1}(MC_t^Z(i))^{1-\psi_{ez}}(Z_t(i))^{\frac{1-\psi_{ez}}{\psi_{ez}}}N_t(i)^{-\frac{1-\psi_{ez}}{\psi_{ez}}}\\
MC_t^Z(i) &= MC_t^Z = \left((1-\alpha_{ez})W_t^{1-\psi_{ez}}+\alpha_{ez}(P_t^E)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}.
\end{aligned}
$$

The Lagrange multiplier of an individual final output firm i does not depend on its own quantities of labor demanded, so that all final output firms have the same multiplier $MC_t^Z(i) = MC_t^Z$.

Price Setting The objective of each final output producing firm is to maximise its nominal profits

$$
DIV_t^Z(i) = P_t(i)Z_t(i) - \left\{\tau_t^Z\left(W_tN_t(i)+P_t^EE_t^z(i)\right)\right\} \quad\Leftrightarrow\quad div_t^Z = \left(1-mc_t^Z\right)Z_t. \tag{A.47}
$$

With probability ϕ_z a firm is stuck with its previous-period price indexed to a composite of previous-period inflation and steady state inflation so that

$$
P_t(i) = \begin{cases} P_t^{\#}(i) & \textit{with probability: } 1-\phi_z\\ P_{t-1}(i)\left((\Pi_{ss})^{1-\xi_z}(\Pi_{t-1})^{\xi_z}\right) & \textit{with probability: } \phi_z \end{cases}
$$

where $\xi_z \in [0,1]$ is the weight attached to previous period inflation. Consider a firm who can reset its price in the current period $P_t(i) = P_t^{\#}(i)$ and who is then stuck with its price until future period $t+s$. The price in this case would be $P_{t+s}(i) = P_t^{\#}(i)\left[(\Pi_{ss})^{s(1-\xi_z)}(P_{t+s-1}/P_{t-1})^{\xi_z}\right]$. Final output firms solve

$$
\max_{P_t^{\#}(i)} E_t\sum_{s=0}^{\infty}(\phi_Z)^s\beta^{t+s}\frac{\lambda_{u,t+s}}{\lambda_{u,t}}\left[P_{t+s}(i)Z_{t+s|t}(i) - MC_{t+s}^Z Z_{t+s|t}(i)\right]
$$

subject to the above derived demand constraint and assuming that a firm z always meets the demand for its good at the current price. $Z_{t+s|t}(i)$ denotes the final output supplied in period $t+s$ by a firm i that last reset its price in period t. If one substitutes the demand schedule and $P_{t+s}(i)$ into the objective function one obtains

$$
\begin{aligned}
\max_{P_t^{\#}(i)} E_t\sum_{s=0}^{\infty}(\phi_Z)^s\beta^{t+s}\frac{\lambda_{u,t+s}}{\lambda_{u,t}}&\left[\left(P_t^{\#}(i)\right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left(\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right]\right)^{1-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left[\left(\frac{1}{P_{t+s}}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}Z_{t+s}\right]\right.\\
&\left. - MC_{t+s}^Z\left[\left(P_t^{\#}(i)\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\left(\frac{\left[(\Pi_{ss})^{s(1-\xi_z)}\left(\frac{P_{t+s-1}}{P_{t-1}}\right)^{\xi_z}\right]}{P_{t+s}}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}Z_{t+s}\right]\right].
\end{aligned}
$$

Taking the derivative with respect to $P_t^{\#}(i)$ delivers the familiar price inflation schedule (A.48)

$$\frac{f_t^{Z,1}}{f_t^{Z,2}}\mathcal{M}_z \quad = \quad \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_z}}}{1-\phi_Z}\right]^{1-\mathcal{M}_z} \tag{A.48}$$

$$f_t^{Z,1} \quad = \quad Z_t mc_t^Z + \phi_Z \beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}}{\Pi_{ss}}\right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z-1}} f_{t+1}^{Z,1}\right] \tag{A.49}$$

$$f_t^{Z,2} \quad = \quad N_t + \phi_Z \beta E_t\left[\left(\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\right)\left(\frac{\Pi_{t+1}}{\Pi_{t+1}^{CPI}}\right)\left(\frac{\Pi_{t+1}}{\Pi_{ss}^{Z}}\right)^{\frac{1}{\mathcal{M}_Z-1}} f_{t+1}^{Z,2}\right] \tag{A.50}$$

$$\zeta_t^Z \quad = \quad \frac{\Pi_t}{(\Pi_{ss})^{1-\xi_z}(\Pi_{t-1})^{\xi_z}} \tag{A.51}$$

$$\mathcal{D}_t^Z \quad = \quad (1-\phi_Z)\left(\frac{1-\phi_Z\left(\zeta_t^Z\right)^{\frac{1}{\mathcal{M}_z-1}}}{1-\phi_Z}\right)^{\mathcal{M}_z} + \phi_Z\left(\zeta_t^Z\right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}}\mathcal{D}_{t-1}^Z. \tag{A.52}$$

Aggregation implies $\int_0^1 Z_t(i)di = \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} Z_t di = Z_t\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} di$ where we define price dispersion as $\mathcal{D}_t^Z \equiv \int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} di$ which can be written recursively

Energy Import Sector Each energy import good $E_t^j, j\in\{z,h\}$ that the final output good producer or the household demands is supplied by a perfectly competitive energy importer. Energy import firms buy a homogenous tradeable energy good on the world market from foreign energy exporters at foreign currency energy price $P_t^{E,*}$. One can transform this into domestic currency by multiplying by the *nominal* exchange rate so that $P_t^E \equiv P_t^{E,*}\mathcal{E}_t$.[32] The importers then transform the homogenous good they purchased $E_t^e = X_t^{e,*}$. After the importers have transformed the energy import good they sell it to domestic final output producers. The cost minimisation problem of importers takes the simple form $\min_{E_t^*}\left\{P_t^{E,*}E_t^*\right\} \quad s.t. \quad E_t^* \geq E_t = E_t^z + E_t^c$. The optimality conditions are given by

$$\mathcal{L} = -\left(P_t^{E,*}\mathcal{E}_t E_t^*\right) + P_t^E E_t^*, \quad \frac{\partial\mathcal{L}}{\partial E_t^*} = 0 \Leftrightarrow P_t^{E,*}\mathcal{E}_t = P_t^E \Leftrightarrow P_t^{E,*}\mathcal{E}_t\frac{1}{P_t}\frac{P_t^*}{P_t^*} = \frac{P_t^E}{P_t} \Leftrightarrow p_t^{E,*}\mathcal{Q}_t = p_t^E \tag{A.53}$$

We assume that the global energy export price level follows the exogenous process

$$p_t^{E,*} \quad = \quad \left(p_{ss}^{E,*}\right)^{1-\rho_E}\left(p_{t-1}^{E,*}\right)^{\rho_E}\varepsilon_t^E. \tag{A.54}$$

Non-Energy Export Sector Exports X_t are produced by a perfectly competitive export good firm. They buy a homogenous non-energy export good on the domestic market from final output retailers at domestic-currency price P_t^X. The 'production' of non-energy export goods works via a simple transformation of final output goods into the expenditure components C, X, so that the supply of a specific export good is given by $X_t = Z_t^X$. The objective of each export good firm is to maximise its nominal profits $DIV_t^X = P_t^{EXP}\mathcal{E}_t X_t - P_t^X Z_t^X \Leftrightarrow div_t^X = \left(p_t^{EXP}\mathcal{Q}_t - p_t^X\right)X_t$ which implies

$$\frac{\partial DIV_t^X}{\partial X_t} = \left(p_t^{EXP}\mathcal{Q}_t - p_t^X\right) = 0 \quad \Leftrightarrow p_t^X = p_t^{EXP}\mathcal{Q}_t \tag{A.55}$$

[32] If for example (from the UK's perspective as the domestic economy) the nominal exchange rate was $\mathcal{E}_t = 0.5$ £/\$ and the importer purchases oil on the world market for $P_t^{E,*} = 100\$$ this would correspond to $P_t^E = (100\$) * (0.5£/\$) = 50£$.

Retailers There is a continuum of perfectly competitive retailers defined on the unit interval, who buy final output goods from the final output good packers at price P_t and convert them into differentiated goods representing each expenditure component: non-energy consumption and non-energy export goods. Retailer r in sector N converts goods using the following linear technology $N_t(r) = Z_t^N(r)$, for $N \in \{C, X\}$ where the input $Z_t^N(r)$ is the amount of the final output good bundle Z_t demanded by retail firm r in expenditure sector N and where the final good bundle, Z_t, is defined by its above stated CES aggregator. Each retailer r in sector N chooses its input $Z_t^N(r)$ to maximise profits, taking the price of its output, $P_t^N, N \in \{C, X\}$ and the price of the final output good, P_t as given

$$\max_{Z_t^N(r)} P_t^N Z_t^N(r) - P_t Z_t^N(r)$$

with first-order condition given by

$$\begin{aligned} P_t^N = P_t, \quad N \in \{C, X\} \quad \Leftrightarrow p_t^X &= P_t^X / P_t = 1, && \text{(A.56)} \\ p_t^C &= P_t^C / P_t = 1 && \text{(A.57)} \end{aligned}$$

A.4 Monetary Policy

Monetary policy follows a simple rule for the nominal interest rate which responds to deviations of annual CPI inflation, $\Pi_t^{CPI,a}$, from its target, $\bar{\Pi}^{CPI,a} = 2\%$, and a measure of the output gap, $\tilde{Y}_t$

$$R_t = \bar{R}^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{CPI,a}}{\bar{\Pi}^{CPI,a}} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\tilde{Y}_t \right)^{(1-\theta_R)\theta_Y} \quad \text{(A.58)}$$

$$\begin{aligned} \text{where} \qquad \Pi_t^{CPI} &= \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t && \text{(A.59)} \\ \Pi_t^{CPI,a} &= \frac{P_t^{CPI}}{P_{t-4}^{CPI}} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} \frac{P_{t-1}^{CPI}}{P_{t-2}^{CPI}} \frac{P_{t-2}^{CPI}}{P_{t-3}^{CPI}} \frac{P_{t-3}^{CPI}}{P_{t-4}^{CPI}} = \Pi_t^{CPI} \Pi_t^{CPI,lag1} \Pi_t^{CPI,lag2} \Pi_{t-1}^{CPI,lag2} && \text{(A.60)} \\ \Pi_t^{CPI,lag1} &= \Pi_{t-1}^{CPI} && \text{(A.61)} \\ \Pi_t^{CPI,lag2} &= \Pi_{t-1}^{CPI,lag1}, \quad \text{where} \bar{\Pi}^{CPI,a} = \left(\bar{\Pi}^{CPI} \right)^4 && \text{(A.62)} \\ \tilde{Y}_t &\equiv N_t / N_t^{flex} && \text{(A.63)} \end{aligned}$$

where N_t^{flex} is the level of employment under flexible prices and wages, $\bar{R}$ is the steady state nominal interest rate consistent with steady-state inflation being at target.

A.5 The World Block

The global demand schedule for the bundle of domestic non-energy exports X_t depends on the foreign currency price of domestic non-energy exports, P_t^{EXP}, relative to the world non-energy export price, P_t^{X*}, and on the world trade volume Z_t^*:

$$X_t = \kappa^* \left(\frac{P_t^{EXP}}{P_t^{X*}} \right)^{-\varsigma^*} Y_{ss}^* \quad \Leftrightarrow \quad X_t = \kappa^* \left(\frac{p_t^{EXP}}{p_{ss}^{X*}} \right)^{-\varsigma^*} Y_{ss}^* \quad \text{(A.64)}$$

where the parameter ς^* is the elasticity of substitution between differentiated export goods in the rest of the world. κ^* can be interpreted as a shifter of the world's preference for domestic exports.

A.6 Summary of Model Equations

Unconstrained Households

$$C_{u,t} = \left(\frac{p_t^C}{p_t^{CPI}}\right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{u,t} \tag{A.1}$$

$$E^h_{u,t} = \left(\frac{p_t^E}{p_t^{CPI}}\right)^{-\psi_{ec}} (\alpha_{ec}) CES_{u,t} \tag{A.2}$$

$$p_t^{CPI} = \left[(1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}} \tag{A.3}$$

$$\lambda_{u,t} = (CES_{u,t})^{-\sigma} \tag{A.4}$$

$$mrs_{u,t} = -\frac{U^N_{u,t}}{\lambda_{u,t}/p_t^{CPI}} \tag{A.5}$$

$$U^N_{u,t} = -\chi\left(N^h_{u,t}\right)^{\varphi} \tag{A.6}$$

$$1 = E_t\left[\Lambda_{u,t,t+1}\left(\Pi^{CPI}_{t+1}\right)^{-1}\right] R_t \tag{A.7}$$

$$\Pi_t^{CPI} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t \tag{A.8}$$

$$0 = E_t\left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{1}{\Pi^{CPI}_{t+1}} \left(R_t - \bar{R}^* \frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t} \frac{\Pi_{t+1}}{\Pi^*_{ss}}\right)\right] \tag{A.9}$$

$$\Lambda_{u,t,t+1} \equiv E_t\left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}}\right] \tag{A.10}$$

$$p_t^C C_{u,t} = w_t N^h_{u,t} + \frac{\left(b_t^* \mathcal{Q}_t - \frac{\bar{R}^* b^*_{t-1} \mathcal{Q}_t}{\Pi^*_{ss}} + div_t^F - t_t^F\right)}{1-\omega} - \mathcal{T}_u - p_t^E E^h_{u,t} \tag{A.11}$$

$$div_t^F = div_t^Z \tag{A.12}$$

Constrained Households

$$C_{c,t} = \left(\frac{p_t^C}{p_t^{CPI}}\right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{c,t} \tag{A.13}$$

$$E^h_{c,t} = \left(\frac{p_t^E}{p_t^{CPI}}\right)^{-\psi_{ec}} (\alpha_{ec}) CES_{c,t} \tag{A.14}$$

$$p_t^{CPI} = \left[(1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}} \tag{A.15}$$

$$\lambda_{c,t} = (CES_{c,t})^{-\sigma} \tag{A.16}$$

$$U^N_{c,t} = -\chi\left(N^h_{c,t}\right)^{\varphi} \tag{A.17}$$

$$p_t^C C_{c,t} = w_t N^h_{c,t} + \mathcal{T}_c - p_t^E E^h_{c,t} \tag{A.18}$$

Aggregation, Market Clearing, Definitions

$$t_t^F = t_t^Z \tag{A.19}$$

$$t_t^Z = (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z) \tag{A.20}$$

$$t_t^L = (1-\tau_t^W) w_t^h N_t^h \tag{A.21}$$

$$C_t = \omega C_{c,t} + (1-\omega) C_{u,t} \tag{A.22}$$

$$E_t^h = \omega E^h_{c,t} + (1-\omega) E^h_{u,t} \tag{A.23}$$

$$N_t^h = \omega N^h_{c,t} + (1-\omega) N^h_{u,t} \tag{A.24}$$

$$mrs_t = \omega mrs_{c,t} + (1-\omega) mrs_{u,t} \tag{A.25}$$

$$\Lambda_{t,t+1} = (1-\omega)\Lambda_{u,t,t+1} \tag{A.26}$$

$$b_t = 0 \tag{A.27}$$

$$p_t^C C_t + p_t^X X_t = Z_t \tag{A.28}$$

$$tb_t = p_t^{EXP} \mathcal{Q}_t X_t - p_t^E (E_t^z + E_t^h) = \frac{\bar{R}^* b^*_{t-1} \mathcal{Q}_t}{\Pi^*_{ss}} - b_t^* \mathcal{Q}_t \tag{A.29}$$

$$\Gamma_t = CES_{u,t}/CES_{c,t} \tag{A.30}$$

$$inc_{u,t} = C_{u,t} + p_t^E E^h_{u,t} + \frac{\bar{R}^* b^*_{u,t-1} \mathcal{Q}_t}{\Pi^*_{ss}} - b^*_{u,t} \mathcal{Q}_t \tag{A.31}$$

$$inc_{c,t} = C_{c,t} + p_t^E E^h_{c,t} \tag{A.32}$$

$$\Gamma_t^{inc} = inc_{u,t}/inc_{c,t} \tag{A.33}$$

Labour Unions $$mc_t^W = \tau^W w_t^h \tag{A.34}$$

$$w_t^h = mrs_t \tag{A.35}$$

$$N_{u,t}^h = N_{c,t}^h \tag{A.36}$$

$$div_t^L = \left(w_t - mc_t^W\right) N_t^h \tag{A.37}$$

$$f_t^{W,1} / f_t^{W,2} \mathcal{M}_w = w_t^\# = \left((1-\phi_w (\zeta_t^W)^{\frac{1}{\mathcal{M}_w - 1}}) / (1-\phi_w) \right)^{1-\mathcal{M}_w} \tag{A.38}$$

$$f_t^{W,1} = N_t mc_t^W / w_t + \phi_w \beta E_t \left[\left(U_{u,t+1}^{CES} / U_{u,t}^{CES} \right) \left(\Pi_{t+1}^W / \Pi_{t+1}^{CPI} \right) \left(\Pi_{t+1}^W / \Pi_{ss}^W \right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} f_{t+1}^{W,1} \right] \tag{A.39}$$

$$f_t^{W,2} = N_t + \phi_w \beta E_t \left[\left(U_{u,t+1}^{CES} / U_{u,t}^{CES} \right) \left(\Pi_{t+1}^W / \Pi_{t+1}^{CPI} \right) \left(\Pi_{t+1}^W / \Pi_{ss}^W \right)^{\frac{1}{\mathcal{M}_w - 1}} f_{t+1}^{W,2} \right] \tag{A.40}$$

$$\zeta_t^W = \frac{\Pi_t^W}{(\Pi_{ss}^W)^{1-\xi_W} (\Pi_{t-1}^W)^{\xi_W}} \tag{A.41}$$

$$w_t = \frac{\Pi_t^W}{\Pi_t} w_{t-1} \tag{A.42}$$

$$\mathcal{D}_t^W = (1-\phi_w) \left(\frac{1-\phi_w \left(\zeta_t^W\right)^{\frac{1}{\mathcal{M}_w - 1}}}{1-\phi_w} \right)^{\mathcal{M}_w} + \phi_w \left(\zeta_t^W\right)^{\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}} \mathcal{D}_{t-1}^W. \tag{A.43}$$

Z Firms $$Z_t \mathcal{D}_t^Z = \varepsilon_t^{TFP} \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} (N_t)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_t^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} \tag{A.44}$$

$$w_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t \mathcal{D}_t^Z}{N_t} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{A.45}$$

$$p_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t \mathcal{D}_t^Z}{E_t^z} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP}\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \tag{A.46}$$

$$div_t^Z = \left(1 - mc_t^Z\right) Z_t \tag{A.47}$$

$$\frac{f_t^{Z,1}}{f_t^{Z,2}} \mathcal{M}_z = \left[\frac{1 - (\phi_Z) \left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_Z}}}{1-\phi_Z} \right]^{1-\mathcal{M}_z} \tag{A.48}$$

$$f_t^{Z,1} = Z_t mc_t^Z + \phi_Z \beta E_t \left[\left(U_{u,t+1}^{CES} / U_{u,t}^{CES} \right) \left(\Pi_{t+1} / \Pi_{t+1}^{CPI} \right) \left(\Pi_{t+1} / \Pi_{ss} \right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,1} \right] \tag{A.49}$$

$$f_t^{Z,2} = N_t + \phi_Z \beta E_t \left[\left(U_{u,t+1}^{CES} / U_{u,t}^{CES} \right) \left(\Pi_{t+1} / \Pi_{t+1}^{CPI} \right) \left(\Pi_{t+1} / \Pi_{ss}^Z \right)^{\frac{1}{\mathcal{M}_Z - 1}} f_{t+1}^{Z,2} \right] \tag{A.50}$$

$$\zeta_t^Z = \frac{\Pi_t}{(\Pi_{ss})^{1-\xi_Z} (\Pi_{t-1})^{\xi_Z}} \tag{A.51}$$

$$\mathcal{D}_t^Z = (1-\phi_Z) \left(\frac{1-\phi_Z \left(\zeta_t^Z\right)^{\frac{1}{\mathcal{M}_Z - 1}}}{1-\phi_Z} \right)^{\mathcal{M}_Z} + \phi_Z \left(\zeta_t^Z\right)^{\frac{\mathcal{M}_Z}{\mathcal{M}_Z - 1}} \mathcal{D}_{t-1}^Z \tag{A.52}$$

$$p_t^E = p_t^{E,*} \mathcal{Q}_t \tag{A.53}$$

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E} \left(p_{t-1}^{E,*}\right)^{\rho_E} \varepsilon_t^E \tag{A.54}$$

$$p_t^X = p_t^{EXP} \mathcal{Q}_t \tag{A.55}$$

$$p_t^X = 1 \tag{A.56}$$

$$p_t^C = 1 \tag{A.57}$$

Monetary Policy $$R_t = R^{1-\theta_R} R_{t-1}^{\theta_R} \left(\frac{\Pi_t^{CPI,a}}{\bar{\Pi}^{CPI,a}} \right)^{\frac{(1-\theta_R)\theta_\Pi}{4}} \left(\tilde{Y}_t\right)^{(1-\theta_R)\theta_Y} \tag{A.58}$$

$$\Pi_t^{CPI} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}} \Pi_t \tag{A.59}$$

$$\Pi_t^{CPI,a} = \frac{P_t^{CPI}}{P_{t-4}^{CPI}} = \frac{P_t^{CPI}}{P_{t-1}^{CPI}} \frac{P_{t-1}^{CPI}}{P_{t-2}^{CPI}} \frac{P_{t-2}^{CPI}}{P_{t-3}^{CPI}} \frac{P_{t-3}^{CPI}}{P_{t-4}^{CPI}} = \Pi_t^{CPI} \Pi_t^{CPI,lag1} \Pi_t^{CPI,lag2} \Pi_{t-1}^{CPI,lag2} \tag{A.60}$$

$$\Pi_t^{CPI,lag1} = \Pi_{t-1}^{CPI} \tag{A.61}$$

$$\Pi_t^{CPI,lag2} = \Pi_{t-1}^{CPI,lag1} \tag{A.62}$$

$$\tilde{Y}_t = \frac{L_t}{L_t^{flex}} \tag{A.63}$$

World $$X_t = \kappa^* \left(p_t^{EXP} / p_{ss}^{X*} \right)^{-\varsigma^*} Y_{ss}^* \tag{A.64}$$

Shocks $$\log \varepsilon_t^{TFP} = \rho_{TFP} \log \varepsilon_{t-1}^{TFP} - \varsigma_{TFP} \eta_t^{TFP}, \qquad \eta_t^{TFP} \sim N(0,1) \tag{A.65}$$

$$\log \varepsilon_t^{\mathcal{M}_z} = \rho_{\mathcal{M}_z} \log \varepsilon_{t-1}^{\mathcal{M}_z} - \varsigma_{\mathcal{M}_z} \eta_t^{\mathcal{M}_z}, \qquad \eta_t^{\mathcal{M}_z} \sim N(0,1) \tag{A.66}$$

$$\log \varepsilon_t^E = \varsigma_E \eta_t^E, \qquad \eta_t^E \sim N(0,1). \tag{A.67}$$

A.7 Log-linearisation

Unconstrained Households

$$C_{u,t} = \left(\frac{p_t^C}{p_t^{CPI}}\right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{u,t}$$

$$C_{u,ss}\left(1+\hat{c}_{u,t}\right) = \left(\frac{p_{ss}^C}{p_{ss}^{CPI}}\right)^{-\psi_{ec}} (1-\alpha_{ec}) CES_{u,ss}\left(1-\psi_{ec}(\hat{p}_t^C-\hat{p}_t^{CPI})+\widehat{ces}_{u,t}\right)$$

$$\hat{c}_{u,t} = \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CPI}) \tag{A.1}$$

$$E_{u,t}^h = \left(\frac{p_t^E}{p_t^{CPI}}\right)^{-\psi_{ec}} (\alpha_{ec}) CES_{u,t}$$

$$\hat{e}_{u,t}^h = \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CPI}) \tag{A.2}$$

$$p_t^{CPI} = \left[(1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}}$$

$$(p_{ss}^{CPI})^{1-\psi_{ec}}\hat{p}_t^{CPI} = (1-\alpha_{ec})\left(p_{ss}^C\right)^{1-\psi_{ec}}\hat{p}_t^C + \alpha_{ec}\left(p_{ss}^E\right)^{1-\psi_{ec}}\hat{p}_t^E$$

$$\hat{p}_t^{CPI} = (1-\alpha_{ec})\hat{p}_t^C + \alpha_{ec}\hat{p}_t^E \tag{A.3}$$

$$\lambda_{u,t} = (CES_{u,t})^{-\sigma}$$

$$\hat{\lambda}_{u,t} = -\sigma\widehat{ces}_{u,t} \tag{A.4}$$

$$mrs_{u,t} = \frac{\chi\left(N_{u,t}^h\right)^{\varphi} p_t^{CPI}}{\lambda_{u,t}}$$

$$\widehat{mrs}_{u,t} = \left(\varphi\hat{n}_{u,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{u,t}\right) \tag{A.5}$$

$$U_{u,t}^N = -\chi\left(N_{u,t}^h\right)^{\varphi}$$

$$U_{u,ss}^N\hat{u}_{u,t}^N = -\chi\left(N_{u,ss}^h\right)^{\varphi}\varphi\hat{n}_{u,t}^h$$

$$\hat{u}_{u,t}^N = \varphi\hat{n}_{u,t}^h \tag{A.6}$$

$$1 = E_t\left[\Lambda_{u,t,t+1}\left(\Pi_{t+1}^{CPI}\right)^{-1}\right]R_t$$

$$1 = \left[\Lambda_{u,ss}\left(\Pi_{ss}^{CPI}\right)^{-1}\right]R_{ss}$$

$$1 = \left[\Lambda_{u,ss}\left(\Pi_{ss}^{CPI}\right)^{-1}\right]R_{ss}\left(1+E_t\hat{\Lambda}_{u,t,t+1} - E_t\hat{\pi}_{t+1}^{CPI} + \hat{r}_t\right)$$

$$0 = E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - E_t\hat{\pi}_{t+1}^{CPI} + \hat{r}_t \tag{A.7}$$

$$\Pi_t^{CPI} = \frac{p_t^{CPI}}{p_{t-1}^{CPI}}\Pi_t$$

$$\hat{\pi}_t^{CPI} = \hat{p}_t^{CPI} - \hat{p}_{t-1}^{CPI} + \hat{\pi}_t \tag{A.8}$$

$$1 = E_t\left[\beta\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\frac{1}{\Pi_{t+1}^{CPI}}\left(\bar{R}^*\frac{\mathcal{Q}_{t+1}}{\mathcal{Q}_t}\frac{\Pi_{t+1}}{\Pi_{ss}^*}\right)\right]$$

$$0 = E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} - E_t\hat{\pi}_{t+1}^{CPI} + E_t\hat{\pi}_{t+1} + \hat{q}_{t+1} - \hat{q}_t \tag{A.9}$$

$$\hat{\Lambda}_{u,t,t+1} = E_t\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} \tag{A.10}$$

$$p_t^C C_{u,t} = w_t N_{u,t}^h + \frac{\left(b_t^*\mathcal{Q}_t - \frac{\bar{R}^* b_{t-1}^*\mathcal{Q}_t}{\Pi_{ss}^*} + div_t^F - t_t^F\right)}{1-\omega} - p_t^E E_{u,t}^h - \mathcal{T}_u$$

$$p_{ss}^C C_{u,ss}\left(\hat{p}_t^C + \hat{c}_{u,t}\right) = \frac{\left(b_{ss}^*\mathcal{Q}_{ss}(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*\mathcal{Q}_{ss}}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - p_{ss}^E E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right),$$

$$\mathcal{Q}_{ss} = 1, p_{ss}^C = 1, p_{ss}^E = 1, \hat{p}_t^C = 0$$

$$C_{u,ss}\hat{c}_{u,t} = \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right) \tag{A.11}$$

$$\widehat{div}_t^F = \widehat{div}_t^Z \tag{A.12}$$

Constrained Households

$$C_{c,t} = \left(\frac{p_t^C}{p_t^{CPI}}\right)^{-\psi_{ec}} (1-\alpha_{ec})CES_{c,t}, \quad \Leftrightarrow \quad \hat{c}_{c,t} = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CPI}) \tag{A.13}$$

$$E_{c,t}^h = \left(\frac{p_t^E}{p_t^{CPI}}\right)^{-\psi_{ec}} (\alpha_{ec})CES_{c,t}, \quad \Leftrightarrow \quad \hat{e}_{c,t}^h = \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CPI}) \tag{A.14}$$

$$p_t^{CPI} = \left[(1-\alpha_{ec})\left(p_t^C\right)^{1-\psi_{ec}} + \alpha_{ec}\left(p_t^E\right)^{1-\psi_{ec}}\right]^{\frac{1}{1-\psi_{ec}}}, \quad \Leftrightarrow \quad \hat{p}_t^{CPI} = (1-\alpha_{ec})\hat{p}_t^C + \alpha_{ec}\hat{p}_t^E \tag{A.15}$$

$$\lambda_{c,t} = (CES_{c,t})^{-\sigma}, \quad \Leftrightarrow \quad \hat{\lambda}_{c,t} = -\sigma\widehat{ces}_{c,t} \tag{A.16}$$

$$mrs_{c,t} = -\frac{U_{c,t}^N}{\lambda_{c,t}/p_t^{CPI}}, \quad \Leftrightarrow \quad \widehat{mrs}_{c,t} = \left(\varphi\hat{n}_{c,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{c,t}\right) \tag{A.17}$$

$$\hat{u}_{u,t}^N = \varphi\hat{n}_{u,t}^h \tag{A.18}$$

$$p_t^C C_{c,t} = w_t N_{c,t}^h + \mathcal{T}_c - p_t^E E_{c,t}^h, \quad \Leftrightarrow \quad C_{c,ss}\hat{c}_{c,t} = w_{ss}N_{c,ss}^h\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - E_{c,ss}^h\left(\hat{p}_t^E + \hat{e}_{c,t}^h\right) \tag{A.19}$$

Aggregation, Market Clearing, Definitions

$$t_t^F = t_t^Z, \quad \Leftrightarrow \quad \hat{t}_t^F = \hat{t}_t^Z \tag{A.20}$$

$$t_t^Z = (1-\tau_t^Z)(w_t N_t^h + p_t^E E_t^z), \Leftrightarrow t_{ss}^Z\hat{t}_t^Z = w_{ss}N_{ss}^h(\hat{w}_t + \hat{n}_t^h) - \tau_{ss}^Z w_{ss}N_{ss}^h(\hat{\tau}_t^Z + \hat{w}_t + \hat{n}_t^h) + p_{ss}^E E_{ss}^z(\hat{p}_t^E + \hat{e}_t^z) - \tau_{ss}^Z p_{ss}^E E_{ss}^z(\hat{\tau}_t^Z + \hat{p}_t^E + \hat{e}_t^z) \tag{A.21}$$

$$t_t^L = (1-\tau_t^W)w_t^h N_t^h, \quad \Leftrightarrow \quad t_{ss}^L\hat{t}_t^L = w_{ss}^h N_{ss}^h(\hat{w}_t^h + \hat{n}_t^h) - \tau_{ss}^W w_{ss}^h N_{ss}^h(\hat{\tau}_t^W + \hat{w}_t^h + \hat{n}_t^h) \tag{A.22}$$

$$C_t = \omega C_{c,t} + (1-\omega)C_{u,t}, \Leftrightarrow C_{ss}\hat{c}_t = \omega C_{c,ss}\hat{c}_{c,t} + (1-\omega)C_{u,ss}\hat{c}_{u,t}, \quad C_{ss} = C_{u,ss} = C_{c,ss}, \quad \hat{c}_t = \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \tag{A.23}$$

$$E_t^h = \omega E_{c,t}^h + (1-\omega)E_{u,t}^h, \quad \Leftrightarrow \quad E_{ss}^h\hat{e}_t^h = \omega E_{c,ss}^h\hat{e}_{c,t}^h + (1-\omega)E_{u,ss}^h\hat{e}_{u,t}^h \tag{A.24}$$

$$N_t^h = \omega N_{c,t}^h + (1-\omega)N_{u,t}^h, \quad \Leftrightarrow \quad N_{ss}^h\hat{n}_t^h = \omega N_{c,ss}^h\hat{n}_{c,t}^h + (1-\omega)N_{u,ss}^h\hat{n}_{u,t}^h \tag{A.25}$$

$$mrs_t = \omega mrs_{c,t} + (1-\omega)mrs_{u,t}, \quad \Leftrightarrow \quad \widehat{mrs}_t = \omega m\hat{r}s_{c,t} + (1-\omega)m\hat{r}s_{u,t} \tag{A.26}$$

$$\Lambda_{t,t+1} = (1-\omega)\Lambda_{u,t,t+1}, \quad \Leftrightarrow \quad \hat{\Lambda}_{t,t+1} = \hat{\Lambda}_{u,t,t+1} \tag{A.27}$$

$$b_t = 0 \tag{A.28}$$

$$p_t^C C_t + p_t^X X_t = Z_t, \quad \Leftrightarrow \quad C_{ss}\hat{c}_t + X_{ss}\hat{x}_t = Z_{ss}\hat{z}_t \tag{A.29}$$

$$tb_t = p_t^{EXP}\mathcal{Q}_t X_t - p_t^{E,*}\mathcal{Q}_t(E_t^z + E_t^h)$$

$$tb_{ss}\hat{tb}_t = p_{ss}^{EXP}X_{ss}(\hat{p}_t^{EXP} + \hat{q}_t + \hat{x}_t) - p_{ss}^{E,*}E_{ss}^z(\hat{p}_t^{E,*} + \hat{q}_t + \hat{e}_t^z) - p_{ss}^{E,*}E_{ss}^h(\hat{p}_t^{E,*} + \hat{q}_t + \hat{e}_t^h) \tag{A.30}$$

$$\Gamma_t = \frac{CES_{u,t}}{CES_{c,t}}, \quad \Leftrightarrow \quad \hat{\gamma}_t = \widehat{ces}_{u,t} - \widehat{ces}_{c,t} \tag{A.31}$$

$$inc_{u,t} = C_{u,t} + p_t^E E_{u,t}^h + \frac{\bar{R}^* b_{u,t-1}^*\mathcal{Q}_t}{\Pi_{ss}^*} - b_{u,t}^*\mathcal{Q}_t \tag{A.32}$$

$$inc_{c,t} = C_{c,t} + p_t^E E_{c,t}^h \tag{A.33}$$

$$\Gamma_t^{inc} = inc_{u,t}/inc_{c,t} \tag{A.34}$$

Labour Unions

$$mc_t^W = \tau^W mrs_t, \quad \Leftrightarrow \quad \hat{mc}_t^W = \widehat{mrs}_t \tag{A.35}$$

$$\hat{w}_t^h = \widehat{mrs}_t \tag{A.36}$$

$$\hat{n}_{u,t} = \hat{n}_{c,t} \tag{A.37}$$

$$div_t^L = \left(w_t - mc_t^W\right)N_t^h, \quad \Leftrightarrow \quad div_{ss}^L\widehat{div}_t^L = w_{ss}N_{ss}^h(\hat{w}_t + \hat{n}_t^h) - mc_{ss}^W N_{ss}^h(\hat{mc}_t^W + \hat{n}_t^h) \tag{A.38}$$

$$\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w = w_t^\# = \left(\frac{1-\phi_w(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}}{1-\phi_w}\right)^{1-\mathcal{M}_w}, \left(\frac{f_t^{W,1}}{f_t^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}} = \left(\frac{1}{1-\phi_w}\right)\left(1-\phi_w(\zeta_t^W)^{\frac{1}{\mathcal{M}_w-1}}\right)$$

$$\left(\frac{f_{ss}^{W,1}}{f_{ss}^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}} = \left(\frac{1}{1-\phi_w}\right) - \left(\frac{1}{1-\phi_w}\right)\left(\phi_w(\zeta_{ss}^W)^{\frac{1}{\mathcal{M}_w-1}}\right)$$

$$\left(\frac{f_{ss}^{W,1}}{f_{ss}^{W,2}}\mathcal{M}_w\right)^{\frac{1}{1-\mathcal{M}_w}}\left(\left(\frac{1}{1-\mathcal{M}_w}\right)(\hat{f}_t^{w,1} - \hat{f}_t^{w,2})\right) = -\left(\frac{1}{1-\phi_w}\right)\left(\phi_w(\zeta_{ss}^W)^{\frac{1}{\mathcal{M}_w-1}}\right)\left(\frac{1}{\mathcal{M}_w-1}\hat{\zeta}_t^W\right)$$

$$\zeta_{ss}^W = 1$$

$$\left(\frac{1}{1-\phi_w}\right)(1-\phi_w)\left(\left(\frac{1}{1-\mathcal{M}_w}\right)(\hat{f}_t^{w,1} - \hat{f}_t^{w,2})\right) = -\left(\frac{1}{1-\phi_w}\right)\phi_w\left(\frac{1}{\mathcal{M}_w-1}\hat{\zeta}_t^W\right), \quad \Leftrightarrow \quad (1-\phi_w)\left(\hat{f}_t^{w,1} - \hat{f}_t^{w,2}\right) = \phi_w\left(\hat{\zeta}_t^W\right) \tag{A.39}$$

$$f_t^{W,1} = \frac{1}{w_t} mc_t^W N_t + \phi_w E_t \left[\beta \frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}} (\zeta_{t+1}^W)^{\frac{\mathcal{M}w}{\mathcal{M}w-1}} f_{t+1}^{W,1} \right]$$

$$f_{ss}^{W,1} = \frac{1}{w_{ss}} mc_{ss}^W N_{ss} + \phi_w \left[\beta \frac{\Pi_{ss}^W}{\Pi_{ss}^{CPI}} (\zeta_{ss}^W)^{\frac{\mathcal{M}w}{\mathcal{M}w-1}} f_{ss}^{W,1} \right], \quad N_{ss} = 1, \Pi_{ss}^W = \Pi_{ss}, \zeta_{ss} = 1, \quad f_{ss}^{W,1} = \frac{mc_{ss}^W}{w_{ss}} \frac{1}{(1-\phi_w \beta)}$$

$$f_{ss}^{W,1} \hat{f}_t^{W,1} = \frac{mc_{ss}^W}{w_{ss}} \left(-\hat{w}_t + \hat{mc}_t^W + \hat{n}_t \right) + \phi_w \beta f_{ss}^{W,1} E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{\mathcal{M}_w}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,1} \right]$$

$$\hat{f}_t^{W,1} = (1-\phi_w \beta) \left(\hat{n}_t + \hat{mc}_t^W - \hat{w}_t \right) + \phi_w \beta E_t \left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{\mathcal{M}_w}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,1} \right] \tag{A.40}$$

$$f_t^{W,2} = N_t + \phi_w E_t \left[\frac{\lambda_{u,t+1}}{\lambda_{u,t}} \frac{\Pi_{t+1}^W}{\Pi_{t+1}^{CPI}} (\zeta_{t+1}^W)^{\frac{1}{\mathcal{M}w-1}} f_{t+1}^{W,2} \right]$$

$$f_{ss}^{W,2} = \frac{1}{(1-\phi_w \beta)}, \quad f_{ss}^{W,2} \hat{f}_t^{W,2} = N_{ss} \hat{n}_t + \phi_w \beta f_{ss}^{W,2} \left(E_t \left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{1}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,2} \right] \right)$$

$$\hat{f}_t^{W,2} = (1-\phi_w \beta) \hat{n}_t + \phi_w \beta \left(E_t \left[\hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{1}{\mathcal{M}_w - 1} \right) \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,2} \right] \right) \tag{A.41}$$

$$\zeta_t^W = \frac{\Pi_t^W}{(\Pi_{ss}^W)^{1-\xi_w} (\Pi_{t-1}^W)^{\xi_w}}, \quad \Leftrightarrow \quad \hat{\zeta}_t^W = \left(\hat{\pi}_t^W - \xi_w \hat{\pi}_{t-1}^W \right) \tag{A.42}$$

$$\hat{w}_t = \hat{\pi}_t^W - \hat{\pi}_t + \hat{w}_{t-1} \tag{A.43}$$

$$\hat{d}_t^W = 0 \tag{A.44}$$

Z Firms

$$Z_t \mathcal{D}_t^Z = \varepsilon_t^{TFP} \left((1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} (N_t)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_t^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}}, \quad \left(Z_t \mathcal{D}_t^Z \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} N_t \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} E_t^z \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}}, \quad \mathcal{D}_{ss}^Z = 1, \varepsilon_{ss}^{TFP} = 1, N_{ss} = 1$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}} (\hat{z}_t) \left(\frac{\psi_{ez}-1}{\psi_{ez}} \right) = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{n}_t \right) \left(\frac{\psi_{ez}-1}{\psi_{ez}} \right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{e}_t^z \right) \left(\frac{\psi_{ez}-1}{\psi_{ez}} \right)$$

$$(Z_{ss})^{\frac{\psi_{ez}-1}{\psi_{ez}}} (\hat{z}_t) = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{n}_t \right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (E_{ss}^z)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{e}_t^z \right)$$

$$\hat{z}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\frac{1}{Z_{ss}} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{n}_t \right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(\frac{E_{ss}^z}{Z_{ss}} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{e}_t^z \right)$$

$$\hat{z}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} (1-\alpha_{ez})^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{n}_t \right) + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} (\alpha_{ez})^{\frac{\psi_{ez}-1}{\psi_{ez}}} \left(\hat{\varepsilon}_t^{TFP} + \hat{e}_t^z \right)$$

$$\hat{z}_t = \hat{\varepsilon}_t^{TFP} + (1-\alpha_{ez}) \hat{n}_t + \alpha_{ez} \hat{e}_t^z \tag{A.45}$$

$$w_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t \mathcal{D}_t^Z}{N_t} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}, \quad \Leftrightarrow \quad w_{ss} = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_{ss}^Z}{\tau_{ss}^Z} \left(\frac{Z_{ss}}{N_{ss}} \right)^{\frac{1}{\psi_{ez}}}$$

$$w_{ss} \hat{w}_t = (1-\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_{ss}^Z}{\tau_{ss}^Z} \left(\frac{Z_{ss}}{N_{ss}} \right)^{\frac{1}{\psi_{ez}}} \left(\hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}} (\hat{z}_t - \hat{n}_t) + \left(\frac{\psi_{ez}-1}{\psi_{ez}} \hat{\varepsilon}_t^{TFP} \right) \right)$$

$$\hat{w}_t = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}} (\hat{z}_t - \hat{n}_t) + \left(\frac{\psi_{ez}-1}{\psi_{ez}} \right) \hat{\varepsilon}_t^{TFP} \tag{A.46}$$

$$p_t^E = (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \frac{mc_t^Z}{\tau_t^Z} \left(\frac{Z_t \mathcal{D}_t^Z}{E_t^z} \right)^{\frac{1}{\psi_{ez}}} \left(\varepsilon_t^{TFP} \right)^{\frac{\psi_{ez}-1}{\psi_{ez}}}, \quad \Leftrightarrow \quad \hat{p}_t^E = \hat{mc}_t^Z - \hat{\tau}_t^Z + \frac{1}{\psi_{ez}} (\hat{z}_t - \hat{e}_t^z) + \left(\frac{\psi_{ez}-1}{\psi_{ez}} \right) \hat{\varepsilon}_t^{TFP} \tag{A.47}$$

$$div_t^Z = \left(1 - mc_t^Z \right) Z_t, \quad \Leftrightarrow \quad div_{ss}^Z \widehat{div}_t^Z = Z_{ss} \hat{z}_t - mc_{ss}^Z Z_{ss} (\hat{mc}_t^Z + \hat{z}_t) \tag{A.48}$$

$$\frac{f_t^{Z,1}}{f_t^{Z,2}}\mathcal{M}_z = \left[\frac{1-(\phi_Z)\left(\zeta_t^Z\right)^{\frac{-1}{1-\mathcal{M}_z}}}{1-\phi_Z}\right]^{1-\mathcal{M}_z}, \quad \Leftrightarrow \quad \hat{\zeta}_t^Z = (1-\phi_Z)/\phi_Z\left(\hat{f}_t^{Z,1}-\hat{f}_t^{Z,2}\right) \tag{A.49}$$

$$f_t^{Z,1} = mc_t^Z Z_t + \phi_Z E_t\left[\frac{\lambda_{u,t+1}}{\lambda_{u,t}}\frac{\Pi_{t+1}}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^Z\right)^{\frac{\mathcal{M}_z}{\mathcal{M}_z-1}} f_{t+1}^{Z,1}\right]$$

$$\hat{f}_t^{Z,1} = (1-\phi_z\beta)\left(\hat{z}_t+\hat{mc}_t^Z\right)+\phi_Z\beta E_t\left[\hat{\lambda}_{u,t+1}-\hat{\lambda}_{u,t}+\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{CPI}+\left(\frac{\mathcal{M}_z}{\mathcal{M}_z-1}\right)\hat{\zeta}_{t+1}^Z+\hat{f}_{t+1}^{Z,1}\right] \tag{A.50}$$

$$f_t^{Z,2} = Z_t + \phi_Z E_t\left[\frac{U_{u,t+1}^{CES}}{U_{u,t}^{CES}}\frac{\Pi_{t+1}}{\Pi_{t+1}^{CPI}}\left(\zeta_{t+1}^Z\right)^{\frac{1}{\mathcal{M}_z-1}} f_{t+1}^{Z,2}\right]$$

$$\hat{f}_t^{Z,2} = (1-\phi_Z\beta)\hat{z}_t+\phi_Z\beta\left(E_t\left[\hat{\lambda}_{u,t+1}-\hat{\lambda}_{u,t}+\hat{\pi}_{t+1}-\hat{\pi}_{t+1}^{CPI}+\left(\frac{1}{\mathcal{M}_Z-1}\right)\hat{\zeta}_{t+1}^Z+\hat{f}_{t+1}^{Z,2}\right]\right) \tag{A.51}$$

$$\zeta_t^Z = \Pi_t/(\Pi_{ss})^{1-\xi_Z}(\Pi_{t-1})^{\xi_Z}, \quad \hat{\zeta}_t^Z = \hat{\pi}_t, \xi_z = 0 \tag{A.52}$$

$$\hat{d}_t^Z = 0 \tag{A.53}$$

$$p_t^E = p_t^{E,*}\mathcal{Q}_t, \quad \Leftrightarrow \quad \hat{p}_t^E = \hat{p}_t^{E,*}+\hat{q}_t \tag{A.54}$$

$$p_t^{E,*} = \left(p_{ss}^{E,*}\right)^{1-\rho_E}\left(p_{t-1}^{E,*}\right)^{\rho_E}\varepsilon_t^E, \quad \Leftrightarrow \quad \hat{p}_t^{E,*} = \rho_E\hat{p}_{t-1}^{E,*}+\hat{\varepsilon}_t^E \tag{A.55}$$

$$p_t^X = p_t^{EXP}\mathcal{Q}_t, \quad \Leftrightarrow \quad 0 = \hat{p}_t^{EXP}+\hat{q}_t \tag{A.56}$$

$$p_t^X = 1, p_t^C = 1, \quad \Leftrightarrow \quad \hat{p}_t^X = 0 \tag{A.57}$$

$$\hat{p}_t^C = 0 \tag{A.58}$$

Monetary Policy and World

$$\hat{r}_t = \theta_R\hat{r}_{t-1}+(1-\theta_R)\left(\theta_\Pi/4\hat{\pi}_t^{CPI,a}+\theta_Y\hat{y}_t\right) \tag{A.59}$$

$$\hat{\pi}_t^{CPI} = \hat{p}_t^{CPI}-\hat{p}_{t-1}^{CPI}+\hat{\pi}_t \tag{A.60}$$

$$\hat{\pi}_t^{CPI,a} = \hat{\pi}_t^{CPI}+\hat{\pi}_t^{CPI,lag1}+\hat{\pi}_t^{CPI,lag2}+\hat{\pi}_{t-1}^{CPI,lag2} \tag{A.61}$$

$$\hat{\pi}_t^{CPI,lag1} = \hat{\pi}_{t-1}^{CPI} \tag{A.62}$$

$$\hat{\pi}_t^{CPI,lag2} = \hat{\pi}_{t-1}^{CPI,lag1} \tag{A.63}$$

$$\hat{y}_t = \hat{n}_t - \hat{n}_t^{flex} \tag{A.64}$$

$$X_t = \kappa^*\left(\frac{p_t^{EXP}}{p_{ss}^{X*}}\right)^{-\varsigma^*}Y_{ss}^*, \quad \hat{x}_t = -\varsigma^*\hat{p}_t^{EXP} \tag{A.65}$$

Shocks

$$\hat{\varepsilon}_t^{TFP} = \rho_{TFP}\hat{\varepsilon}_{t-1}^{TFP}-\varsigma_{TFP}\eta_t^{TFP} \tag{A.66}$$

$$\hat{\varepsilon}_t^{\mathcal{M}_z} = \rho_{\mathcal{M}_z}\hat{\varepsilon}_{t-1}^{\mathcal{M}_z}-\varsigma_{\mathcal{M}_z}\eta_t^{\mathcal{M}_z} \tag{A.67}$$

$$\hat{\varepsilon}_t^E = \varsigma_E\eta_t^E \tag{A.68}$$

A.8 Reduce the loglinear system

Unconstrained HH loglinear system

Step 1: $\hat{p}_t^C = 0$, take out $\hat{\Lambda}$, $\hat{\lambda}_{u,t}$, $\hat{u}^N$

$$
\begin{aligned}
\hat{c}_{u,t} &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^C - \hat{p}_t^{CPI}), \qquad \hat{c}_{u,t} = \widehat{ces}_{u,t} + \psi_{ec}\hat{p}_t^{CPI} \\
\hat{e}_{u,t}^h &= \widehat{ces}_{u,t} - \psi_{ec}(\hat{p}_t^E - \hat{p}_t^{CPI}) \\
\hat{p}_t^{CPI} &= (1-\alpha_{ec})\hat{p}_t^C + \alpha_{ec}\hat{p}_t^E, \hat{p}_t^C = 0, \qquad \hat{p}_t^{CPI} = \alpha_{ec}\hat{p}_t^E \\
\widehat{mrs}_{u,t} &= \left(\varphi\hat{n}_{u,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{u,t}\right) \\
-\sigma\widehat{ces}_{u,t} &= -\mathbf{E}_t[\sigma\widehat{ces}_{u,t+1}] - \mathbf{E}_t\hat{\pi}_{t+1}^{CPI} + \hat{r}_t \\
\hat{\pi}_t^{CPI} &= \hat{p}_t^{CPI} - \hat{p}_{t-1}^{CPI} + \hat{\pi}_t \\
-\sigma\widehat{ces}_{u,t} &= -\mathbf{E}_t[\sigma\widehat{ces}_{u,t+1}] - \mathbf{E}_t\hat{\pi}_{t+1}^{CPI} + \mathbf{E}_t\hat{\pi}_{t+1} + \hat{q}_{t+1} - \hat{q}_t \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 2

$$
\begin{aligned}
\hat{c}_{u,t} &= \widehat{ces}_{u,t} + \psi_{ec}\hat{p}_t^{CPI}, \qquad \hat{p}_t^{CPI} = \alpha_{ec}\hat{p}_t^E, \qquad \widehat{ces}_{u,t} = \mathbf{E}_t[\widehat{ces}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\hat{\pi}_{t+1}^{CPI}\right) \\
\hat{\pi}_t^{CPI} &= \hat{p}_t^{CPI} - \hat{p}_{t-1}^{CPI} + \hat{\pi}_t = \alpha_{ec}(\hat{p}_t^E - \hat{p}_{t-1}^E) + \hat{\pi}_t \\
\widehat{ces}_{u,t} &= \mathbf{E}_t[\widehat{ces}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{q}_{t+1} - \hat{q}_t - \mathbf{E}_t\hat{\pi}_{t+1}^{CPI} + \mathbf{E}_t\hat{\pi}_{t+1}\right) \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 3

$$
\begin{aligned}
\widehat{ces}_{u,t} &= \hat{c}_{u,t} - \psi_{ec}\alpha_{ec}\hat{p}_t^E, \quad \hat{e}_{u,t}^h = \hat{c}_{u,t} - \psi_{ec}\alpha_{ec}\hat{p}_t^E - \psi_{ec}(1-\alpha_{ec})\hat{p}_t^E \\
\hat{c}_{u,t} - \psi_{ec}\alpha_{ec}\hat{p}_t^E &= \mathbf{E}_t[\hat{c}_{u,t+1} - \psi_{ec}\alpha_{ec}\hat{p}_{t+1}^E] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}^{CPI}\right]\right) \\
\hat{c}_{u,t} - \psi_{ec}\alpha_{ec}\hat{p}_t^E &= \mathbf{E}_t[\hat{c}_{u,t+1} - \psi_{ec}\alpha_{ec}\hat{p}_{t+1}^E] - \frac{1}{\sigma}\left(\hat{q}_{t+1} - \hat{q}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}^{CPI}\right] + \mathbf{E}_t\hat{\pi}_{t+1}\right) \\
C_{u,ss}\hat{c}_{u,t} &= \frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^F - t_{ss}^F\hat{t}_t^F\right)}{1-\omega} \\
&\quad + w_{ss}N_{u,ss}^h\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - E_{u,ss}^h\left(\hat{p}_t^E + \hat{e}_{u,t}^h\right)
\end{aligned}
$$

Step 4

$$
\begin{aligned}
\hat{c}_{u,t} &= \mathbf{E}_t[\hat{c}_{u,t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}^{CPI}\right]\right) + \psi_{ec}\alpha_{ec}\left(\hat{p}_t^E - \mathbf{E}_t\hat{p}_{t+1}^E\right) \\
\hat{r}_t - \mathbf{E}_t\hat{\pi}_{t+1} &= \mathbf{E}_t\hat{q}_{t+1} - \hat{q} \\
\hat{c}_{u,t} &= \frac{1}{C_{u,ss}}\frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t) + div_{ss}^F\widehat{div}_t^Z - t_{ss}^Z\hat{t}_t^Z\right)}{1-\omega} \\
&\quad + w_{ss}\frac{N_{u,ss}^h}{C_{u,ss}}\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - \frac{E_{u,ss}^h}{C_{u,ss}}\left(\hat{c}_{u,t} + \hat{p}_t^E(1-\psi_{ec})\right)
\end{aligned}
$$

Note that $E^h_{u,ss} = 0$ if $\alpha_{ec} = 0$. Use the the Z profit conditions and the tax equations

$$
\begin{aligned}
div^Z_t - t^Z_t &= Z_t - w_t N_t - p^E_t E^z_t + p^{EXP}_t \mathcal{Q}_t X_t - p^X_t X_t \\
div^Z_{ss}\widehat{div}^Z_t - t^Z_{ss}\hat{t}^Z_t &= Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)
\end{aligned}
$$

Constrained HH loglinear system

Step 1: take out $\hat{\lambda}_{c,t}$, $\hat{p}^C_t = 0$

$$
\begin{aligned}
\hat{c}_{c,t} &= \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}^C_t - \hat{p}^{CPI}_t) \\
\hat{e}^h_{c,t} &= \widehat{ces}_{c,t} - \psi_{ec}(\hat{p}^E_t - \hat{p}^{CPI}_t) \\
\hat{p}^{CPI}_t &= (1-\alpha_{ec})\hat{p}^C_t + \alpha_{ec}\hat{p}^E_t \\
\hat{\lambda}_{c,t} &= -\sigma\widehat{ces}_{c,t} \\
\widehat{mrs}_{c,t} &= \left(\varphi\hat{n}^h_{c,t} + \hat{p}^{CPI}_t - \hat{\lambda}_{c,t}\right) \\
\hat{u}^N_{u,t} &= \varphi\hat{n}^h_{u,t} \\
C_{c,ss}\hat{c}_{c,t} &= w_{ss}N^h_{c,ss}\left(\hat{w}_t + \hat{n}^h_{c,t}\right) - E^h_{c,ss}\left(\hat{p}^E_t + \hat{e}^h_{c,t}\right)
\end{aligned}
$$

which implies

$$
\begin{aligned}
\widehat{ces}_{c,t} &= \hat{c}_{c,t} - \psi_{ec}\alpha_{ec}\hat{p}^E_t \\
\hat{e}^h_{c,t} &= \widehat{ces}_{c,t} - \psi_{ec}\hat{p}^E_t(1-\alpha_{ec}) \\
\hat{c}_{c,t} &= w_{ss}\frac{N^h_{c,ss}}{C_{c,ss}}\left(\hat{w}_t + \hat{n}^h_{c,t}\right) - \frac{E^h_{c,ss}}{C_{c,ss}}\left(\hat{c}_{c,t} + \hat{p}^E_t(1-\psi_{ec})\right)
\end{aligned}
$$

- note that $E^h_{c,ss} = 0$ if $\alpha_{ec} = 0$

Remaining HH loglinear system

Step 1

$$
\begin{aligned}
\hat{\gamma}_t &= \widehat{ces}_{u,t} - \widehat{ces}_{c,t} \\
t^Z_{ss}\hat{t}^Z_t &= w_{ss}N^h_{ss}(\hat{w}_t + \hat{n}^h_t) - \tau^Z_{ss}w_{ss}N^h_{ss}(\hat{\tau}^Z_t + \hat{w}_t + \hat{n}^h_t) + p^E_{ss}E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) \\
&\quad - \tau^Z_{ss}p^E_{ss}E^z_{ss}(\hat{\tau}^Z_t + \hat{p}^E_t + \hat{e}^z_t) \\
\hat{c}_t &= \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t} \\
\hat{e}^h_t &= \omega\frac{E^h_{c,ss}}{E^h_{ss}}\hat{e}^h_{c,t} + (1-\omega)\frac{E^h_{u,ss}}{E^h_{ss}}\hat{e}^h_{u,t} \\
\hat{n}^h_t &= \omega\frac{N^h_{c,ss}}{N^h_{ss}}\hat{n}^h_{c,t} + (1-\omega)\frac{N^h_{u,ss}}{N^h_{ss}}\hat{n}^h_{u,t} \\
\hat{p}^{CPI}_t &= \hat{p}^E_t\left(\omega\alpha_{ec} + (1-\omega)\alpha_{ec}\right) \\
\hat{z}_t &= \frac{C_{ss}}{Z_{ss}}\hat{c}_t + \frac{X_{ss}}{Z_{ss}}\hat{x}_t \\
tb_{ss}\hat{tb}_t &= X_{ss}(\underbrace{\hat{p}^{EXP}_t + \hat{q}_t}_{=0} + \hat{x}_t) - E^z_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^z_t) - E^h_{ss}(\hat{p}^{E,*}_t + \hat{q}_t + \hat{e}^h_t), \quad \hat{p}^E_t = \hat{p}^{E,*}_t + \hat{q}_t
\end{aligned}
$$

Log-linearisation of the Consumption Gap with Energy in Production and Consumption

$$\Gamma_t = \underbrace{\frac{w_t N_t + \frac{1}{1-\omega}\left(Z_t - w_t N_t - p_t^E E_t^z\right)}{w_t N_t}}_{\text{income gap}} + \underbrace{\frac{1}{1-\omega}\frac{p_t^E E_t^z + p_t^E E_t^h - X_t}{w_t N_t}}_{\text{U HHs debt variation}}$$

$\Gamma_t = 1 + \frac{1}{1-\omega}\frac{Z_t - w_t N_t - p_t^E E_t^z}{w_t N_t} + \frac{1}{1-\omega}\frac{Z_t}{w_t N_t}\frac{p_t^E E_t^z + p_t^E E_t^h - X_t}{Z_t}$ but $\frac{Z_t - w_t N_t - p_t^E E_t^z}{w_t N_t} = \frac{\frac{Z_t - w_t N_t - p_t^E E_t^z}{w_t N_t + p_t^E E_t^z}}{\frac{w_t N_t}{w_t N_t + p_t^E E_t^z}} = \frac{\mathcal{M}_t - 1}{\frac{1}{1+\frac{p_t^E E_t^z}{w_t N_t}}} = \frac{\mathcal{M}_t - 1}{\frac{1}{1+\frac{p_t^E E_t^z}{w_t N_t}}} = \frac{\mathcal{M}_t - 1}{\frac{1}{1+\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}}}}$,

since $\frac{E_t^z}{N_t} = \frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{-\psi_{ez}}$.

Besides $\frac{p_t^E E_t^h - p_t^X X_t}{Z_t} = \frac{p_t^E E_t^h}{C_t}\frac{C_t}{Z_t} - \frac{p_t^X X_t}{Z_t} = \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p_t^E\right)^{1-\psi_{ec}}\frac{C_t}{Z_t} + \frac{C_t}{Z_t} - \frac{C_t}{Z_t} - \frac{X_t}{Z_t} = \frac{C_t}{Z_t} + \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p_t^E\right)^{1-\psi_{ec}}\frac{C_t}{Z_t} - 1$ since $\frac{E_t^h}{C_t} = \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(\frac{p_t^E}{p_t^C}\right)^{-\psi_{ec}} = \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p_t^E\right)^{-\psi_{ec}}$.

Then $\Gamma_t = 1 + \frac{1}{1-\omega}\frac{\mathcal{M}_t - 1}{\frac{1}{1+\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}}}} + \frac{1}{1-\omega}\frac{p_t^E E_t^z}{w_t N_t} + \frac{1}{1-\omega}\frac{Z_t}{w_t N_t}\left(\frac{C_t}{Z_t} + \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p_t^E\right)^{1-\psi_{ec}}\frac{C_t}{Z_t} - 1\right)$

$\Gamma_t = 1 + \frac{1}{1-\omega}\frac{\mathcal{M}_t - 1}{\frac{1}{1+\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}}}} + \frac{1}{1-\omega}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}} + \frac{1}{1-\omega}\frac{\mathcal{M}_t}{\frac{1}{1+\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}}}}\left(\frac{C_t}{Z_t} + \frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p_t^E\right)^{1-\psi_{ec}}\frac{C_t}{Z_t} - 1\right)$

since $\frac{Z_t}{w_t N_t} = \frac{\frac{Z_t}{w_t N_t + p_t^E E_t^z}}{\frac{w_t N_t}{w_t N_t + p_t^E E_t^z}} = \frac{\mathcal{M}_t}{\frac{1}{1+\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p_t^E}{w_t}\right)^{1-\psi_{ez}}}}$.

Log-linearize

$$\begin{aligned}
\Gamma e^{\hat{\gamma}_t} = {} & 1 + \frac{1}{1-\omega}\left(\mathcal{M}e^{\mu_t} - 1 + \mathcal{M}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{\mu_t + (1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)} - \frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{(1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)}\right) \\
& + \frac{1}{1-\omega}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{(1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)} + \\
& \frac{1}{1-\omega}\left(\mathcal{M}\frac{C}{Z}e^{(\hat{c}_t - \hat{z}_t + \hat{\mu}_t)} + \mathcal{M}\frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p^E\right)^{1-\psi_{ec}}\frac{C}{Z}e^{(1-\psi_{ec})\hat{p}_t^E + \hat{\mu}_t + \hat{c}_t - \hat{z}_t} - \mathcal{M}e^{\hat{\mu}_t}\right) + \\
& \frac{1}{1-\omega}\left(\mathcal{M}\frac{C}{Z}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{\left(\hat{c}_t - \hat{z}_t + \hat{\mu}_t + (1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)\right)}\right. \\
& + \mathcal{M}\frac{\alpha_{ec}}{1-\alpha_{ec}}\left(p^E\right)^{1-\psi_{ec}}\frac{C}{Z}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{(1-\psi_{ec})\hat{p}_t^E + \hat{\mu}_t + \hat{c}_t - \hat{z}_t + (1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)} \\
& \left. - \mathcal{M}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{p^E}{w}\right)^{1-\psi_{ez}} e^{\hat{\mu}_t + (1-\psi_{ez})\left(\hat{p}_t^E - \hat{w}_t\right)}\right)
\end{aligned}$$

$$\begin{aligned}
\Gamma\hat{\gamma}_t \quad = \quad & \underbrace{\frac{1}{1-\omega}\left(M_1\mathcal{M}\hat{\mu}_t + M_2\left(\mathcal{M}-1\right)\left(1-\psi_{ez}\right)\left(\hat{p}_t^E - \hat{w}_t\right)\right)}_{\text{Income gap}} + \\
& \underbrace{\frac{B_1}{1-\omega}\left(1-\psi_{ez}\right)\left(\hat{p}_t^E - \hat{w}_t\right) + \frac{\mathcal{M}}{1-\omega}\left(B_2\left(\widehat{ces}_t - \hat{x}_t\right) + B_3\hat{\mu}_t + B_4\left(1 - \frac{C}{Z}\psi_{ec}\right)\hat{p}_t^E\right)}_{\text{Borrowing}}
\end{aligned} \tag{A.69}$$

where we introduced the auxiliary terms for the income gap

$$M_1 \equiv \frac{wN + E^z}{wN} > 1, \qquad M_2 \equiv \frac{E^z}{wN} = M_1 - 1 > 0$$

and for the borrowing term

$$B_1 \equiv M_2\left(1 + \mathcal{M}\left(\frac{CES}{Z} - 1\right)\right) > 0, \quad B_2 \equiv M_1\frac{CES}{Z}\frac{X}{Z} > 0, \quad B_3 \equiv M_1\left(\frac{CES}{Z} - 1\right) < 0, \quad B_4 \equiv M_1\frac{E^h}{Z} > 0$$

Note that CES/Z = (1-X/Z)=0.75 in our calibration. This implies that an increase in markups

$$\frac{\partial \hat{\gamma}_t}{\partial \hat{\mu}_t} = \frac{\mathcal{M}}{1-\omega}(M_1 + B_3) = \frac{\mathcal{M}M_1}{1-\omega}\left(1 + \frac{CES}{Z} - 1\right) > 0$$

increases the consumption gap. Moreover, defining the energy price wage ratio as $\hat{pw}_t \equiv (\hat{p}_t^E - \hat{w}_t)$ we have

$$\frac{\partial \hat{\gamma}_t}{\partial \hat{pw}_t} = \frac{1-\psi_{ez}}{1-\omega}(M_2(\mathcal{M}-1) + B_1) > 0 \quad \text{as long as} \quad \psi_{ez} < 1.$$

Finally, we have that an increase in energy prices increases the consumption gap

$$\frac{\partial \hat{\gamma}_t}{\partial \hat{p}_t^E} = \frac{\mathcal{M}}{1-\omega}B_4\left(1 - \frac{C}{Z}\psi_{ec}\right) > 0 \qquad \text{as long as} \quad \psi_{ec} < \frac{Z}{C} = 1/(1-0.25) = 1.3333 \text{ in our calibration.}$$

Marginal Rate of substitution

$$\widehat{mrs}_t = \omega \hat{mrs}_{c,t} + (1-\omega)\hat{mrs}_{u,t},\ \widehat{mrs}_{u,t} = \left(\varphi \hat{n}_{u,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{u,t}\right),\ \widehat{mrs}_{c,t} = \left(\varphi \hat{n}_{c,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{c,t}\right)$$

$$\widehat{mrs}_t = \omega\left(\varphi \hat{n}_{c,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{c,t}\right) + (1-\omega)\left(\varphi \hat{n}_{u,t}^h + \hat{p}_t^{CPI} - \hat{\lambda}_{u,t}\right), \quad \hat{p}_t^{CPI} = \alpha_{ec}\hat{p}_t^E,\ \hat{p}_t^{CPI} = \alpha_{ec}\hat{p}_t^E$$

$$\hat{\lambda}_{u,t} = -\sigma\widehat{ces}_{u,t}, \quad \hat{c}_{u,t} = \widehat{ces}_{u,t} + \psi_{ec}\hat{p}_t^{CPI}$$

$$\widehat{mrs}_t = \omega\left(\varphi \hat{n}_{c,t}^h + \sigma\hat{c}_{c,t}\right) + (1-\omega)\left(\varphi \hat{n}_{u,t}^h + \sigma\hat{c}_{u,t}\right) + \omega\left(\alpha_{ec}\hat{p}_t^E(1-\sigma\psi_{ec})\right) + (1-\omega)\left(\alpha_{ec}\hat{p}_t^E(1-\sigma\psi_{ec})\right)$$

$$\widehat{mrs}_t = \varphi\hat{n}_t^h + \sigma\hat{c}_t + (\omega\alpha_{ec} + (1-\omega)\alpha_{ec})\,\hat{p}_t^E(1-\sigma\psi_{ec})$$

Aggregate demand (AD) non-policy block The household block can be combined into the non-policy aggregate demand block

$$\widehat{mrs}_t = \varphi\hat{n}_t^h + \sigma\hat{c}_t + \alpha_{ec}\hat{p}_t^E(1-\sigma\psi_{ec}) \tag{A.70}$$

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}_{t+1}^{CPI}\right]\right) + \psi_{ec}\alpha_{ec}\left(\hat{p}_t^E - \mathbf{E}_t\hat{p}_{t+1}^E\right) \tag{A.71}$$

$$\hat{r}_t = \mathbf{E}_t\hat{q}_{t+1} - \hat{q} + \mathbf{E}_t\hat{\pi}_{t+1} \tag{A.72}$$

$$\begin{aligned}\hat{c}_{u,t} = &\frac{1}{C_{u,ss}}\frac{\left(b_{ss}^*(\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*}(\hat{b}_{t-1}^* + \hat{q}_t)\right)}{1-\omega} \\ &+ \frac{1}{C_{u,ss}}\frac{\left(Z_{ss}\hat{z}_t - w_{ss}N_{ss}(\hat{w}_t + \hat{n}_t) - E_{ss}^z(\hat{p}_t^E + \hat{e}_t^z) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)\right)}{1-\omega} \\ &+ w_{ss}\frac{N_{u,ss}^h}{C_{u,ss}}\left(\hat{w}_t + \hat{n}_{u,t}^h\right) - \frac{E_{u,ss}^h}{C_{u,ss}}\left(\hat{c}_{u,t} + \hat{p}_t^E(1-\psi_{ec})\right)\end{aligned} \tag{A.73}$$

$$\hat{c}_{c,t} = w_{ss}\frac{N_{c,ss}^h}{C_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}^h\right) - \frac{E_{c,ss}^h}{C_{c,ss}}\left(\hat{c}_{c,t} + \hat{p}_t^E(1-\psi_{ec})\right) \tag{A.74}$$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{A.75}$$

$$\hat{c}_t = \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t}, \quad \hat{c}_{u,t} = \hat{c}_t + \omega\hat{\gamma}_t \tag{A.76}$$

$$\hat{e}_t^h = \omega\frac{E_{c,ss}^h}{E_{ss}^h}\hat{e}_{c,t}^h + (1-\omega)\frac{E_{u,ss}^h}{E_{ss}^h}\hat{e}_{u,t}^h \tag{A.77}$$

$$\hat{n}_t^h = \omega \frac{N_{c,ss}^h}{N_{ss}^h} \hat{n}_{c,t}^h + (1-\omega) \frac{N_{u,ss}^h}{N_{ss}^h} \hat{n}_{u,t}^h = \omega \hat{n}_{c,t}^h + (1-\omega) \hat{n}_{u,t}^h \tag{A.78}$$

$$\hat{p}_t^{CPI} = \hat{p}_t^E \left(\omega \alpha_{ec} + (1-\omega) \alpha_{ec}\right) \tag{A.79}$$

$$\hat{z}_t = \frac{C_{ss}}{Z_{ss}} \hat{c}_t + \frac{X_{ss}}{Z_{ss}} \hat{x}_t \tag{A.80}$$

$$\widehat{tb}_t = \frac{X_{ss}}{tb_{ss}} \hat{x}_t - \hat{p}_t^E \left(\frac{E_{ss}^z + E_{ss}^h}{tb_{ss}} \right) - \frac{E_{ss}^z}{tb_{ss}} \hat{e}_t^z - \frac{E_{ss}^h}{tb_{ss}} \hat{e}_t^h \tag{A.81}$$

IS equations features the consumption gap

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega \Delta \hat{\gamma}_{t+1}] - \frac{1}{\sigma} \left(\hat{r}_t - \mathbf{E}_t \left[\hat{\pi}_{t+1}^{CPI} \right] \right) - \psi_{ec} \alpha_{ec} \Delta \mathbf{E}_t \hat{p}_{t+1}^E$$

- if energy enters the unconstrained consumption basket, then $\alpha_{ec} > 0$
- if energy is close to a Leontief, hard to substitute good, ψ_{ec} is very low, $(1 - \sigma \psi_{ec})$ is large, the effect of energy in the C basket is then aggravated

Show in a seperate step that the C gap is affected by energy, even if $\alpha_{ec} = 0$

$$\begin{aligned}
\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t}, \qquad \hat{\gamma}_t = \Bigg(& \frac{1}{C_{u,ss}} \frac{\left(b_{ss}^* (\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*} (\hat{b}_{t-1}^* + \hat{q}_t) \right)}{1-\omega} \\
& + \frac{1}{C_{u,ss}} \frac{\left(Z_{ss} \hat{z}_t - w_{ss} N_{ss} (\hat{w}_t + \hat{n}_t) - E_{ss}^z (\hat{p}_t^E + \hat{e}_t^z) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t) \right)}{1-\omega} \\
& + w_{ss} \frac{N_{u,ss}^h}{C_{u,ss}} \left(\hat{w}_t + \hat{n}_{u,t}^h \right) - \frac{E_{u,ss}^h}{C_{u,ss}} \left(\hat{c}_{u,t} + \hat{p}_t^E (1 - \psi_{ec}) \right) \Bigg) \\
& - \left(w_{ss} \frac{N_{c,ss}^h}{C_{c,ss}} \left(\hat{w}_t + \hat{n}_{c,t}^h \right) - \frac{E_{c,ss}^h}{C_{c,ss}} \left(\hat{c}_{c,t} + \hat{p}_t^E (1 - \psi_{ec}) \right) \right)
\end{aligned}$$

If $\alpha_{ec} = \alpha_{x,ec} = 0$, then $E_{c,ss}^h = E_{u,ss}^h = 0$

$$\begin{aligned}
\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t}, \qquad \hat{\gamma}_t = \Bigg(& \frac{1}{C_{u,ss}} \frac{\left(b_{ss}^* (\hat{b}_t^* + \hat{q}_t) - \frac{\bar{R}^* b_{ss}^*}{\Pi_{ss}^*} (\hat{b}_{t-1}^* + \hat{q}_t) \right)}{1-\omega} \\
& + \frac{1}{C_{u,ss}} \frac{\left(Z_{ss} \hat{z}_t - w_{ss} N_{ss} (\hat{w}_t + \hat{n}_t) - E_{ss}^z (\hat{p}_t^E + \hat{e}_t^z) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t) \right)}{1-\omega} \\
& + w_{ss} \frac{N_{u,ss}^h}{C_{u,ss}} \left(\hat{w}_t + \hat{n}_{u,t}^h \right) \Bigg) - \left(w_{ss} \frac{N_{c,ss}^h}{C_{c,ss}} \left(\hat{w}_t + \hat{n}_{c,t}^h \right) \right)
\end{aligned}$$

Energy shocks matter even if $\alpha_{ec} = \alpha_{ec} = 0$ since they directly affect firm profits, and indirectly affect labour demand, and hence household labour income, which affects unconstrained and constrained households differently.

Reduce the Union and Firm loglinear system to get the AS block

Combine Union Equations to Wage Philips Curve

$$\hat{n}_t = \hat{n}_{u,t} = \hat{n}_{c,t}, \qquad \hat{w}_t^h = \widehat{mrs}_t, \qquad \hat{\zeta}_t^W = \frac{1-\phi_w}{\phi_w}\left(\hat{f}_t^{w,1} - \hat{f}_t^{w,2}\right)$$

$$\hat{f}_t^{W,1} = (1-\phi_w\beta)\left(\hat{n}_t + \widehat{mrs}_t - \hat{w}_t\right) + \phi_w\beta E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{\mathcal{M}_w}{\mathcal{M}_w - 1}\right)\hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,1}\right]$$

$$\hat{f}_t^{W,2} = (1-\phi_w\beta)\hat{n}_t + \phi_w\beta\left(E_t\left[\hat{\lambda}_{u,t+1} - \hat{\lambda}_{u,t} + \hat{\pi}_{t+1}^W - \hat{\pi}_{t+1}^{CPI} + \left(\frac{1}{\mathcal{M}_w - 1}\right)\hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,2}\right]\right)$$

$$\hat{\zeta}_t^W = \hat{\pi}_t^W, \xi_w = 0, \ \hat{w}_t = \hat{\pi}_t^W - \hat{\pi}_t + \hat{w}_{t-1}$$

Next

$$\hat{f}_t^{W,1} - \hat{f}_t^{W,2} = (1-\phi_w\beta)\left(\widehat{mrs}_t - \hat{w}_t\right) + \phi_w\beta E_t\left[\hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{W,1} - \hat{f}_{t+1}^{W,2}\right]$$

$$\frac{\phi_w}{1-\phi_w}\hat{\zeta}_t^W = \hat{f}_t^{w,1} - \hat{f}_t^{w,2}, \ \hat{\zeta}_{t+1}^W + \frac{\phi_w}{1-\phi_w}\hat{\zeta}_{t+1}^W = \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{w,1} - \hat{f}_{t+1}^{w,2}, \ \frac{1}{1-\phi_w}\hat{\zeta}_{t+1}^W = \hat{\zeta}_{t+1}^W + \hat{f}_{t+1}^{w,1} - \hat{f}_{t+1}^{w,2}$$

$$\frac{\phi_w}{1-\phi_w}\hat{\zeta}_t^W = (1-\phi_w\beta)\left(\widehat{mrs}_t - \hat{w}_t\right) + \phi_w\beta E_t\left[\frac{1}{1-\phi_w}\hat{\zeta}_{t+1}^W\right]$$

So we have the wage inflation system as follows

$$\hat{\pi}_t^W = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{A.82}$$

$$\hat{\pi}_t^W = \frac{(1-\phi_w\beta)(1-\phi_w)}{\phi_w}\left(\hat{w}_t^h - \hat{w}_t\right) + \beta E_t\left[\hat{\pi}_{t+1}^W\right] \tag{A.83}$$

Energy shocks affect the wage PC via the wage markup wedge $\hat{w}_t^h - \hat{w}_t$ where

$$\hat{w}_t^h = \widehat{mrs}_t = \varphi\hat{n}_t^h + \sigma\hat{c}_t + (\omega\alpha_{ec} + (1-\omega)\alpha_{ec})\,\hat{p}_t^E(1-\sigma\psi_{ec})$$

Combine Z Firm Equations to Domestic Z Price Philips Curve

$$\hat{z}_t = \hat{\varepsilon}_t^{TFP} + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}_t^z \tag{A.84}$$

$$\hat{mc}_t^Z = \hat{w}_t + \hat{\varepsilon}_t^{\mathcal{M}_z} - \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{n}_t) - \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{TFP} \tag{A.85}$$

$$\hat{mc}_t^Z = \hat{p}_t^E + \hat{\varepsilon}_t^{\mathcal{M}_z} - \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{e}_t^z) - \left(\frac{\psi_{ez}-1}{\psi_{ez}}\right)\hat{\varepsilon}_t^{TFP}, \quad \hat{\tau}_t^Z = \hat{\varepsilon}_t^{\mathcal{M}_z} \tag{A.86}$$

$$\hat{\pi}_t = \frac{(1-\phi_z\beta)(1-\phi_z)}{\phi_z}\left(\hat{mc}_t^Z\right) + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{A.87}$$

$$\hat{p}_t^E = \hat{p}_t^{E,*} + \hat{q}_t \tag{A.88}$$

$$\hat{p}_t^{E,*} = \rho_E\hat{p}_{t-1}^{E,*} + \hat{\varepsilon}_t^E \tag{A.89}$$

$$\hat{q}_t = -\hat{p}_t^{EXP}$$

Monetary Policy and World

$$\hat{r}_t = \theta_R\hat{r}_{t-1} + (1-\theta_R)\left(\theta_\Pi/4\hat{\pi}_t^{CPI,a} + \theta_Y(\hat{n}_t - \hat{n}_t^{flex})\right) \tag{A.90}$$

$$\hat{\pi}_t^{CPI,a} = \hat{\pi}_t^{CPI} + \hat{\pi}_{t-1}^{CPI} + \hat{\pi}_{t-2}^{CPI} + \hat{\pi}_{t-3}^{CPI} \tag{A.91}$$

$$\hat{x}_t = \varsigma^*\hat{q}_t \tag{A.92}$$

A.9 Summary of log-linear model: AD, AS, World and Policy Block

Household Demand

$$\hat{c}_t = \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) - \psi_{ec}\alpha_{ec}\Delta\mathbf{E}_t\hat{p}^E_{t+1} \tag{A.93}$$

$$\begin{aligned}\hat{c}_{u,t} = {} & \frac{N^h_{u,ss}}{E^h_{u,ss} + C_{u,ss}}\left(\hat{w}_t + \hat{n}^h_{u,t}\right) - \alpha_{ec}\left(1-\psi_{ec}\right)\hat{p}^E_t + \frac{\left(b^*_{ss}(\hat{b}^*_t + \hat{q}_t) - \frac{\bar{R}^* b^*_{ss}}{\Pi^*_{ss}}(\hat{b}^*_{t-1} + \hat{q}_t)\right)}{\left(E^h_{u,ss} + C_{u,ss}\right)(1-\omega)} \\ & + \frac{\left(Z_{ss}\left(\hat{\varepsilon}^{TFP}_t + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}^z_t\right) - (\hat{w}_t + \hat{n}_t) - E^z_{ss}(\hat{p}^E_t + \hat{e}^z_t) + X_{ss}(\hat{x}_t) - X_{ss}(\hat{x}_t)\right)}{\left(E^h_{u,ss} + C_{u,ss}\right)(1-\omega)}\end{aligned} \tag{A.94}$$

$$\hat{c}_{c,t} = \frac{N^h_{c,ss}}{C_{c,ss} + E^h_{c,ss}}\left(\hat{w}_t + \hat{n}_{c,t}\right) - \alpha_{ec}\left(1-\psi_{ec}\right)\hat{p}^E_t, \quad E^h_{c,ss}/(E^h_{c,ss} + C_{c,ss}) = \alpha_{ec} \tag{A.95}$$

$$\hat{\gamma}_t = \hat{c}_{u,t} - \hat{c}_{c,t} \tag{A.96}$$

$$\hat{c}_t = \omega\hat{c}_{c,t} + (1-\omega)\hat{c}_{u,t}, \quad \Leftrightarrow \quad \hat{c}_{u,t} = \hat{c}_t + \omega\hat{\gamma}_t \tag{A.97}$$

Market Clearing

$$\hat{w}^h_t = \widehat{mrs}_t = \varphi\hat{n}^h_t + \sigma\hat{c}_t + \left(\omega\alpha_{ec} + (1-\omega)\alpha_{ec}\right)\hat{p}^E_t(1-\sigma\psi_{ec}) \tag{A.98}$$

$$\hat{n}_t = \hat{n}_{u,t} = \hat{n}_{c,t} \tag{A.99}$$

$$\hat{c}_t = \frac{Z_{ss}}{C_{ss}}\left(\hat{\varepsilon}^{TFP}_t + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}^z_t\right) - \frac{X_{ss}}{C_{ss}}\hat{x}_t \tag{A.100}$$

Wage and Price Setting

$$\hat{\pi}^W_t = \frac{(1-\phi_w\beta)(1-\phi_w)}{\phi_w}\left(\hat{w}^h_t - \hat{w}_t\right) + \beta E_t\left[\hat{\pi}^W_{t+1}\right], \quad \hat{\pi}^W_t = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t \tag{A.101}$$

$$\hat{\pi}_t = \frac{(1-\phi_z\beta)(1-\phi_z)}{\phi_z}\left(\hat{mc}^Z_t\right) + \beta E_t\left[\hat{\pi}_{t+1}\right] \tag{A.102}$$

$$\hat{mc}^Z_t = \hat{w}_t + \hat{\varepsilon}^{\mathcal{M}_z}_t - \frac{1}{\psi_{ez}}(\alpha_{ez}\hat{e}^z_t - \alpha_{ez}\hat{n}_t) - \hat{\varepsilon}^{TFP}_t \tag{A.103}$$

$$\hat{mc}^Z_t = \hat{p}^E_t + \hat{\varepsilon}^{\mathcal{M}_z}_t - \frac{1}{\psi_{ez}}((1-\alpha_{ez})\hat{n}_t - (1-\alpha_{ez})\hat{e}^z_t) - \hat{\varepsilon}^{TFP}_t \tag{A.104}$$

World

$$\hat{x}_t = \varsigma^*\hat{q}_t \tag{A.105}$$

$$\hat{r}_t = \mathbf{E}_t\hat{q}_{t+1} - \hat{q} + \mathbf{E}_t\hat{\pi}_{t+1} \tag{A.106}$$

$$\hat{p}^E_t = \hat{p}^{E,*}_t + \hat{q}_t \tag{A.107}$$

$$\hat{p}^{E,*}_t = \rho_E\hat{p}^{E,*}_{t-1} + \hat{\varepsilon}^E_t \tag{A.108}$$

Monetary Policy

$$\hat{r}_t = \theta_R\hat{r}_{t-1} + (1-\theta_R)\left((\theta_\Pi/4)\hat{\pi}^{CPI,a}_t + \theta_Y(\hat{n}_t - \hat{n}^{flex}_t)\right), \qquad \hat{\pi}^{CPI,a}_t \equiv \sum_{j=0}^{3}\hat{\pi}^{CPI}_{t-j} \tag{A.109}$$

$$\hat{\pi}^{CPI}_t = \hat{\pi}_t + \alpha_{ec}\Delta\hat{p}^E_t \tag{A.110}$$

Shocks to (i) TFP, (ii) price markups and (iii) global energy prices

$$\hat{\varepsilon}^{TFP}_t = \rho_{TFP}\hat{\varepsilon}^{TFP}_{t-1} - \varsigma_{TFP}\eta^{TFP}_t, \quad \hat{\varepsilon}^{\mathcal{M}_z}_t = \rho_{\mathcal{M}_z}\hat{\varepsilon}^{\mathcal{M}_z}_{t-1} - \varsigma_{\mathcal{M}_z}\eta^{\mathcal{M}_z}_t, \quad \hat{\varepsilon}^E_t = \varsigma_E\eta^E_t$$

A.9.1 IS Derivation

Combine consumption IS with aggregate resource constraint

$$
\begin{aligned}
\hat{c}_t &= \mathbf{E}_t[\hat{c}_{t+1}] + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) - \psi_{ec}\alpha_{ec}\Delta\mathbf{E}_t\hat{p}^E_{t+1} \\
0 &= \mathbf{E}_t[\Delta\hat{c}_{t+1}] - \frac{1}{\sigma}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) + \mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \left(\alpha_{ec}\psi_{ec}\mathbf{E}_t\left[\Delta\hat{p}^E_{t+1}\right]\right) \\
0 &= \frac{C_{ss}}{Z_{ss}}\mathbf{E}_t[\Delta\hat{c}_{t+1}] - \frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) + \frac{C_{ss}}{Z_{ss}}\mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{C_{ss}}{Z_{ss}}\left(\alpha_{ec}\psi_{ec}\mathbf{E}_t\left[\Delta\hat{p}^E_{t+1}\right]\right) \\
\hat{c}_t &= \frac{Z_{ss}}{C_{ss}}\left(\hat{\varepsilon}^{TFP}_t + (1-\alpha_{ez})\hat{n}_t + \alpha_{ez}\hat{e}^z_t\right) - \frac{X_{ss}}{C_{ss}}\varsigma^*\hat{q}_t \\
\frac{C_{ss}}{Z_{ss}}\Delta\hat{c}_{t+1} &= \left(\Delta\hat{\varepsilon}^{TFP}_{t+1} + (1-\alpha_{ez})\Delta\hat{n}_{t+1} + \alpha_{ez}\Delta\hat{e}^z_{t+1}\right) - \frac{X_{ss}}{Z_{ss}}\varsigma^*\Delta\hat{q}_{t+1}
\end{aligned}
$$

to get

$$
\begin{aligned}
0 = {} & \Delta\mathbf{E}_t\hat{\varepsilon}^{TFP}_{t+1} + (1-\alpha_{ez})\Delta\mathbf{E}_t\hat{n}_{t+1} + \alpha_{ez}\Delta\mathbf{E}_t\hat{e}^z_{t+1} - \frac{X_{ss}}{Z_{ss}}\varsigma^*\Delta\mathbf{E}_t\hat{q}_{t+1} - \frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) \\
& + \frac{C_{ss}}{Z_{ss}}\mathbf{E}_t[\omega\Delta\hat{\gamma}_{t+1}] - \frac{C_{ss}}{Z_{ss}}\left(\alpha_{ec}\psi_{ec}\mathbf{E}_t\left[\Delta\hat{p}^E_{t+1}\right]\right)
\end{aligned}
$$

Use the energy demand schedule

$$
\begin{aligned}
\hat{mc}^Z_t &= \hat{p}^E_t + \hat{\varepsilon}^{\mathcal{M}_z}_t - \frac{1}{\psi_{ez}}((1-\alpha_{ez})\hat{n}_t - (1-\alpha_{ez})\hat{e}^z_t) - \hat{\varepsilon}^{TFP}_t \\
\psi_{ez}\left(\hat{mc}^Z_t - \hat{p}^E_t - \hat{\varepsilon}^{\mathcal{M}_z}_t + \hat{\varepsilon}^{TFP}_t\right) &= -(1-\alpha_{ez})\hat{n}_t + (1-\alpha_{ez})\hat{e}^z_t \\
\hat{e}^z_t &= \frac{\psi_{ez}}{1-\alpha_{ez}}\left(\hat{mc}^Z_t - \hat{p}^E_t - \hat{\varepsilon}^{\mathcal{M}_z}_t + \hat{\varepsilon}^{TFP}_t\right) + \hat{n}_t \\
\alpha_{ez}\Delta\hat{e}^z_{t+1} &= \frac{\alpha_{ez}\psi_{ez}}{1-\alpha_{ez}}\left(\Delta\hat{mc}^Z_{t+1} - \Delta\hat{p}^E_{t+1} - \Delta\hat{\varepsilon}^{\mathcal{M}_z}_{t+1} + \Delta\hat{\varepsilon}^{TFP}_{t+1}\right) + \alpha_{ez}\Delta\hat{n}_{t+1}, \quad \hat{mc}^Z_t = -\hat{\mu}^z_t
\end{aligned}
$$

so that

$$
\begin{aligned}
\hat{n}_t = {} & \mathbf{E}_t\left[\hat{n}_{t+1}\right] + \psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(-\mathbf{E}_t\Delta\hat{\mu}^Z_{t+1} - \mathbf{E}_t\Delta\hat{p}^E_{t+1} - \mathbf{E}_t\Delta\hat{\varepsilon}^{\mathcal{M}_z}_{t+1}\right) + \frac{1-\alpha_{ez}+\alpha_{ez}\psi_{ez}}{1-\alpha_{ez}}\Delta\mathbf{E}_t\left[\hat{\varepsilon}^{TFP}_{t+1}\right] \\
& - \frac{X_{ss}}{Z_{ss}}\varsigma^*\mathbf{E}_t\Delta\hat{q}_{t+1} - \frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\left(\hat{r}_t - \mathbf{E}_t\left[\hat{\pi}^{CPI}_{t+1}\right]\right) \\
& + \omega\frac{C_{ss}}{Z_{ss}}\mathbf{E}_t[\Delta\hat{\gamma}_{t+1}] - \alpha_{ec}\frac{C_{ss}}{Z_{ss}}\left(\psi_{ec}\mathbf{E}_t\left[\Delta\hat{p}^E_{t+1}\right]\right).
\end{aligned} \tag{A.111}
$$

Solving this forward, we obtain a dynamic IS curve for employment $\hat{n}_t$, a proxy for 'aggregate demand', broken down by channels

$$
\begin{aligned}
\hat{n}_t = {} & \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\hat{r}_{t+k} - \hat{\pi}^{CPI}_{t+k+1}\right)}_{\text{Inter-temporal substitution (-)}} \quad \underbrace{-\omega\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\substack{\text{Demand effect from} \\ \text{credit constraints (+/-)}}} \quad \underbrace{+\psi_{ez}\frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\hat{p}^E_t + \hat{\mu}^z_t + \hat{\varepsilon}^{\mathcal{M}_z}_t\right)}_{\substack{\text{Intra-temporal substitution} \\ \text{in production (+)}}} \quad \underbrace{+\frac{X_{ss}}{Z_{ss}}\varsigma^*\hat{q}_t}_{\text{terms of trade (+)}} \\
& - \left(\frac{1-\alpha_{ez}+\psi_{ez}\alpha_{ez}}{1-\alpha_{ez}}\right)\hat{\varepsilon}^{TFP}_t \quad \underbrace{+\alpha_{ec}\psi_{ec}\frac{C_{ss}}{Z_{ss}}\hat{p}^E_t}_{\substack{\text{Intra-temporal substitution} \\ \text{in consumption (+)}}}
\end{aligned} \tag{A.112}
$$

Recall that $\hat{\pi}_t^{CPI} = \hat{\pi}_t + \alpha_{ec}\Delta\hat{p}_t^E$ so that for $\alpha_{ec} = 0$ we have $\hat{\pi}_t^{CPI} = \hat{\pi}_t$. Note that we can rewrite the firm's labor and energy demand schedules

$$\begin{aligned} \hat{mc}_t^Z &= \hat{w}_t + \hat{\varepsilon}_t^{\mathcal{M}_z} - \psi_{ez}^{-1}(\alpha_{ez}\hat{e}_t^z - \alpha_{ez}\hat{n}_t) - \hat{\varepsilon}_t^{TFP} \\ \hat{mc}_t^Z &= \hat{p}_t^E + \hat{\varepsilon}_t^{\mathcal{M}_z} - \psi_{ez}^{-1}((1-\alpha_{ez})\hat{n}_t - (1-\alpha_{ez})\hat{e}_t^z) - \hat{\varepsilon}_t^{TFP} \end{aligned}$$

and combine them into

$$-\hat{\mu}_t^Z \equiv \hat{mc}_t^Z = (1-\alpha_{ez})\hat{w}_t + \alpha_{ez}\hat{p}_t^E + \hat{\varepsilon}_t^{\mathcal{M}_z} - \hat{\varepsilon}_t^{TFP}.$$

This implies that the channel related to the intra-temporal substitution in production can be re-written in terms of the difference between real energy prices and real wages

$$\hat{p}_t^E + \hat{\mu}_t^Z + \hat{\varepsilon}_t^{\mathcal{M}_z} = (1-\alpha_{ez})(\hat{p}_t^E - \hat{w}_t) + \hat{\varepsilon}_t^{TFP}.$$

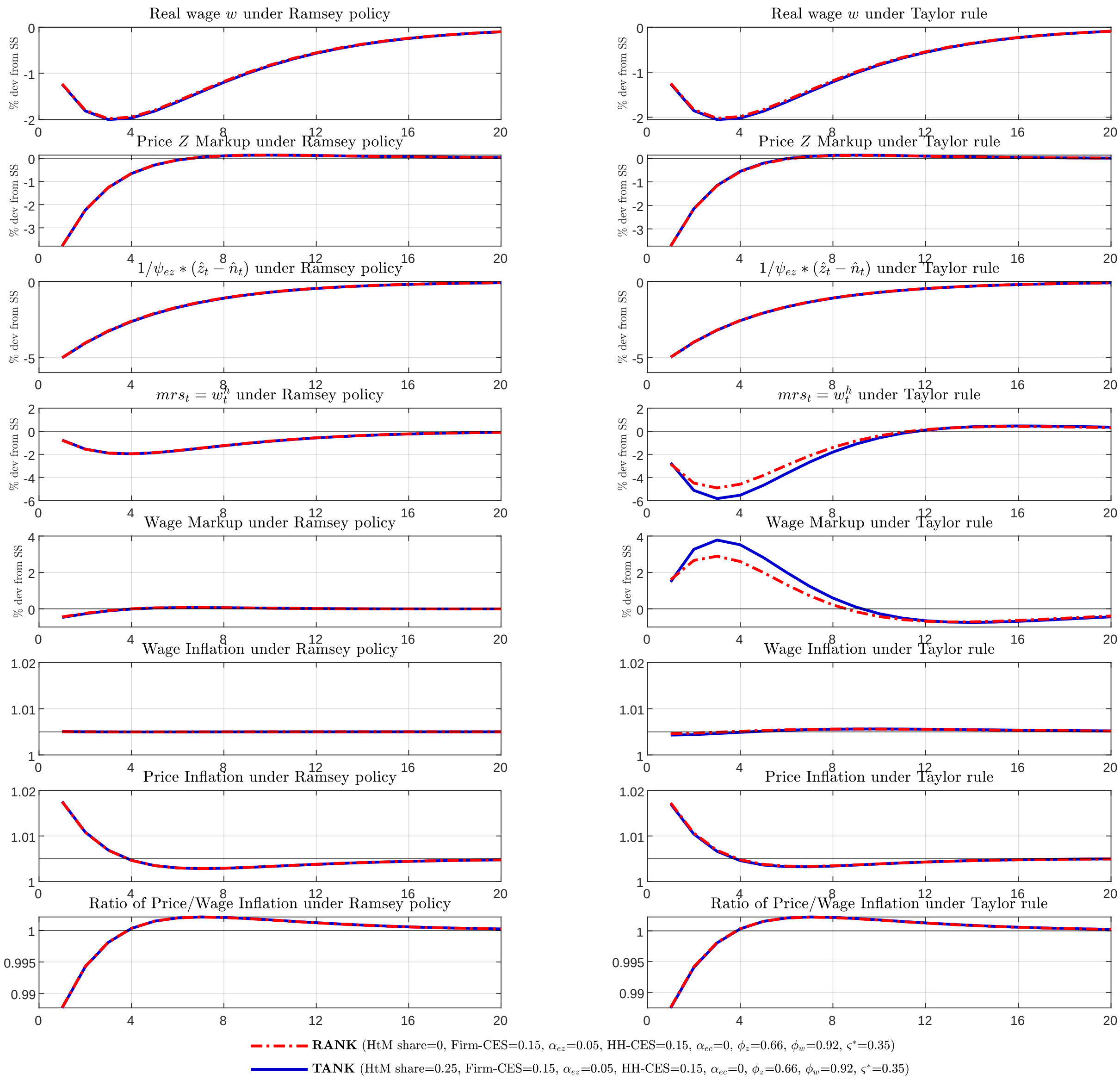
The dynamic IS equation (A.112) can thus be written as

$$\begin{aligned} \hat{n}_t = & \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}\left(\hat{r}_{t+k} - \hat{\pi}_{t+k+1}^{CPI}\right)}_{\text{Inter-temporal substitution (-)}} \quad \underbrace{-\omega\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\substack{\text{Demand effect from} \\ \text{credit constraints (+/-)}}} \quad \underbrace{+\psi_{ez}\alpha_{ez}\left(\hat{p}_t^E - \hat{w}_t\right)}_{\substack{\text{Intra-temporal substitution} \\ \text{in production (+)}}} \quad \underbrace{+\frac{X_{ss}}{Z_{ss}}\varsigma^*\hat{q}_t}_{\text{terms of trade (+)}} \quad -\hat{\varepsilon}_t^{TFP} \\ & \underbrace{+\alpha_{ec}\psi_{ec}\frac{C_{ss}}{Z_{ss}}\hat{p}_t^E}_{\substack{\text{Intra-temporal substitution} \\ \text{in consumption (+)}}} \end{aligned} \tag{A.113}$$

A.10 The Effects of Energy Prices on Price and Wage Markups

In the lower four rows of Figure A.1 we can see that although the Ramsey planner succeeds in stabilising the wage markup more than under Taylor, the outcome for the gross rate of wage inflation $\Pi_t^W = W_t/W_{t-1}$ is still very similar to the Taylor case. Given the high value for $\phi_w = 0.92$, in line with the standard in the literature (Schmitt-Grohe and Uribe (2006)), the New Keynesian Wage Philips curve is 'flat'. This also explains why the responses of the real wage under Taylor and Ramsey are rather similar, since $w_t/w_{t-1} = \Pi_t/\Pi_t^W$ and since the ratio of price to wage inflation is very similar under Ramsey policy and a Taylor-rule, both, in the TANK and RANK case.

FIGURE A.1: Dynamic Responses to a Global Energy Price Shock: Ramsey and Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key components of the wage markup to a 100% increase in the foreign currency price of energy. The right (left) column shows responses for a case in which the central bank follows a Taylor rule (Ramsey policy). The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.

Price Markups Recall the equation that describes the price markup as a function of the final output firm's cost of energy and labour

$$\frac{1}{\mathcal{M}_t^Z} = \frac{MC_t^Z}{P_t} = \left((1 - \alpha_{ez}) \left(\frac{W_t}{P_t} \right)^{1-\psi_{ez}} + \alpha_{ez} \left(\frac{P_t^E}{P_t} \right)^{1-\psi_{ez}} \right)^{\frac{1}{1-\psi_{ez}}}$$

An increase in nominal energy prices P_t^E will also lead to an increase in the 'real' price of energy, p_t^E, due to nominal price stickiness. Given that nominal wages are more sticky than nominal prices, the real wage $w_t \equiv W_t/P_t$ falls. The real increase in energy prices dominates, the real marginal cost thus increases and the price markup $\mathcal{M}_t^Z$ falls.

In the upper four rows of Figure A.1 we can see that the responses under Ramsey (left column) and under Taylor (right column) are fairly similar. However, since the Ramsey-policy planner is more accommodative, he can prevent the fall in the marginal rate of substitution that would otherwise materialise under a Taylor rule. The more accommodative stance under Ramsey policy does not alter the profiles for price markups or real wages.

Wage Markups Recall the equations that pin down the real wage and set $\hat{\varepsilon}_t^{\mathcal{M}_z} = 0$ and $\hat{\varepsilon}_t^{TFP} = 0$ and the equation that pins down the marginal rate of substitution and assume that energy only enters the production function, so that $\alpha_{ec} = 0$

$$\begin{aligned}
\hat{w}_t &= \hat{mc}_t^Z + \frac{1}{\psi_{ez}}(\hat{z}_t - \hat{n}_t). \\
\hat{w}_t^h &= \widehat{mrs}_t = \varphi \hat{n}_t^h + \sigma \hat{c}_t + \alpha_{ec} \hat{p}_t^E (1 - \sigma \psi_{ec}), \quad \hat{w}_t^h = \widehat{mrs}_t = \varphi \hat{n}_t^h + \sigma \hat{c}_t
\end{aligned}$$

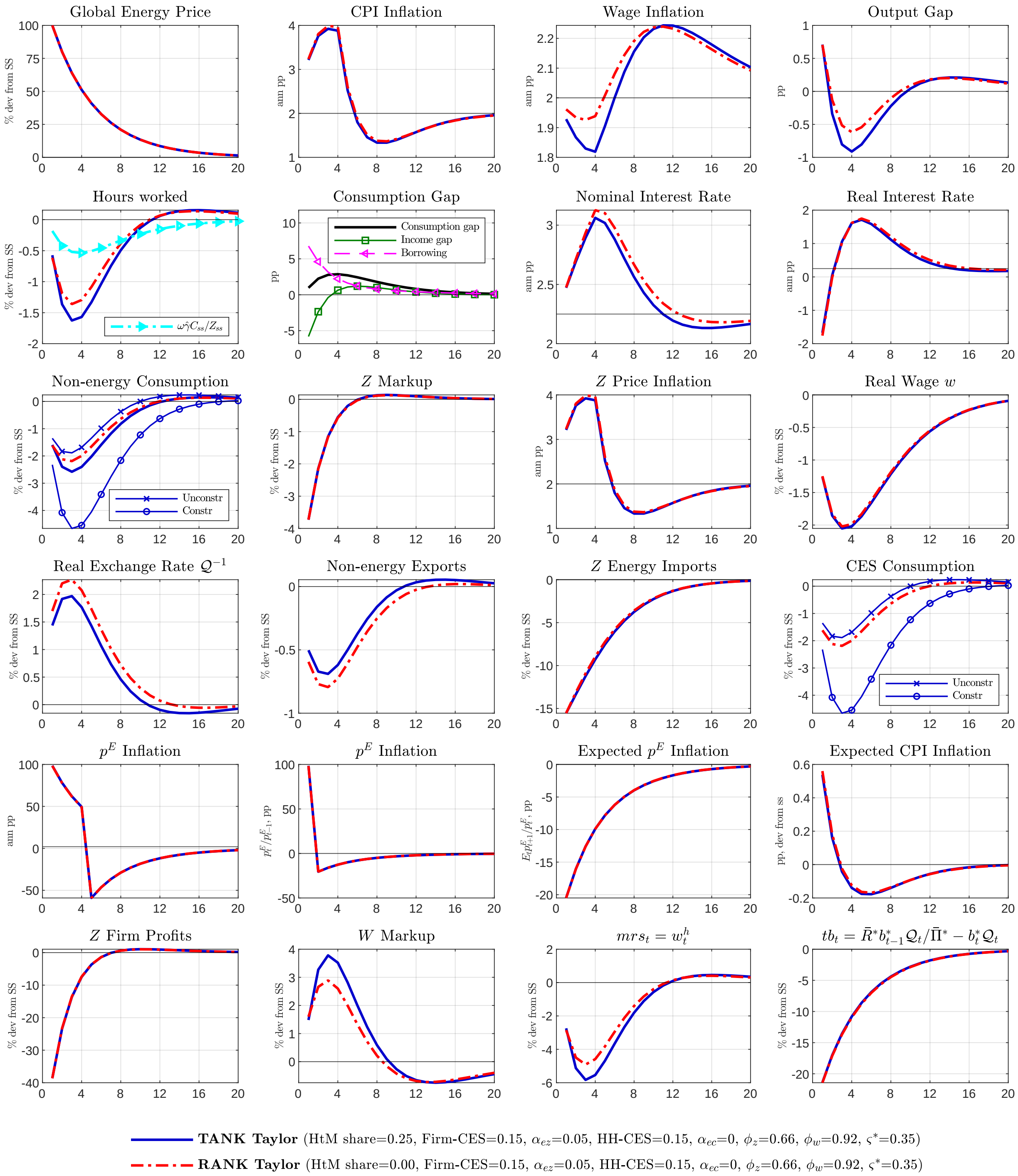
The wage markup is defined as the wedge between the real wage and the mrs

$$\hat{\mu}_t^W \equiv \hat{w}_t - m\hat{r}s_t.$$

B Additional Impulse Response Functions

B.1 Energy shock with energy only in production, Taylor

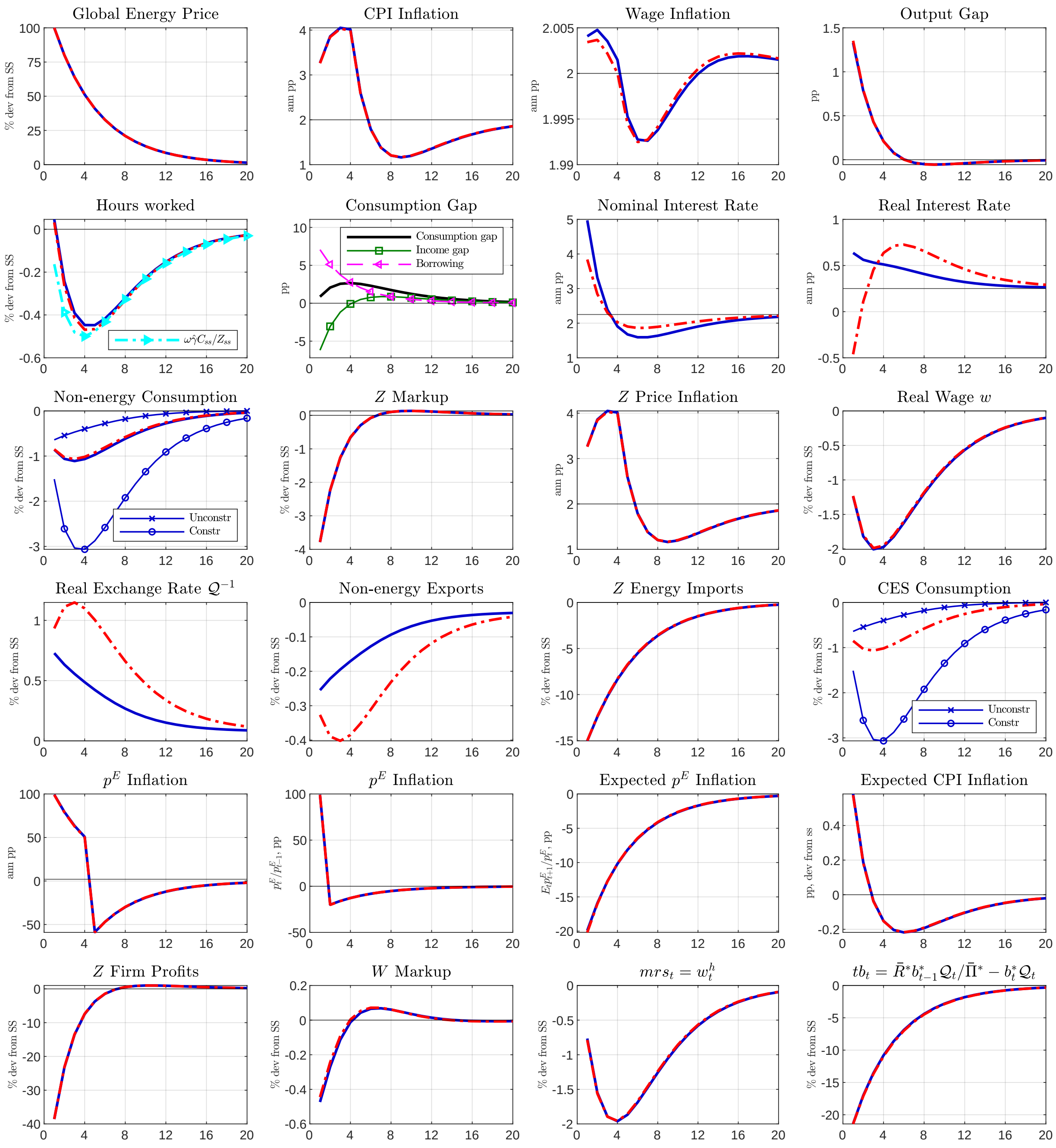
FIGURE B.2: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.

B.2 Energy shock with energy only in production, Ramsey

FIGURE B.3: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



TANK Ramsey (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

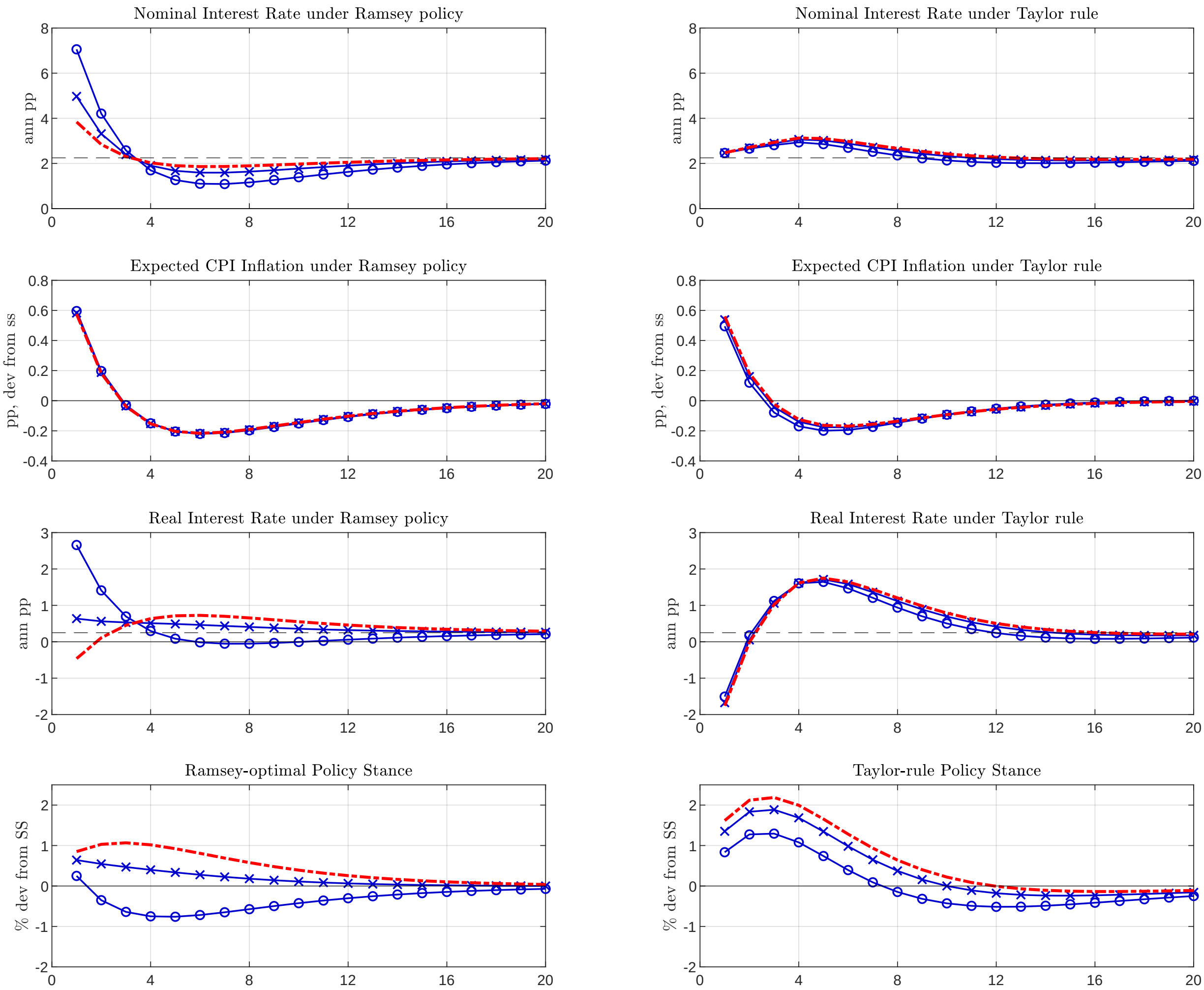
RANK Ramsey (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.

B.3 Ramsey vs Taylor Policy Stance, Energy shock with energy only in production

Figure B.4 presents the IRFs for the nominal interest rate (top row), the expected inflation rate (second row from the top), the real interest rate (third row) and the policy stance (bottom row) over an increasingly larger share of constrained agents, which allows the energy price shock to yield a correspondingly larger fall in households' consumption. It can be seen that the Ramsey planner raises the nominal rate forcefully on impact, while the nominal rate only reaches 3% after 4 quarters or so. The Ramsey planner then cuts the nominal rate below the steady state level of 2.25%. Under a Taylor rule, the nominal rate stays above the steady state. While the real rate rises strongly under Ramsey-policy initially, it also falls below steady state. In the RANK model, Ramsey-optimal monetary policy remains contractionary throughout the period of higher energy prices in order to counteract inflation. Meanwhile, in the TANK, optimal policy under a HtM weight of $\omega = 0.25$ is less contractionary. Under a higher share of constrained HtM agents, $\omega = 0.5$, the optimal policy stance can even be accomodative, as can be seen in the circled blue line in the lower left panel in Figure B.4.

FIGURE B.4: Dynamic Responses to a Global Energy Price Shock: Policy with Stronger Credit Constraints

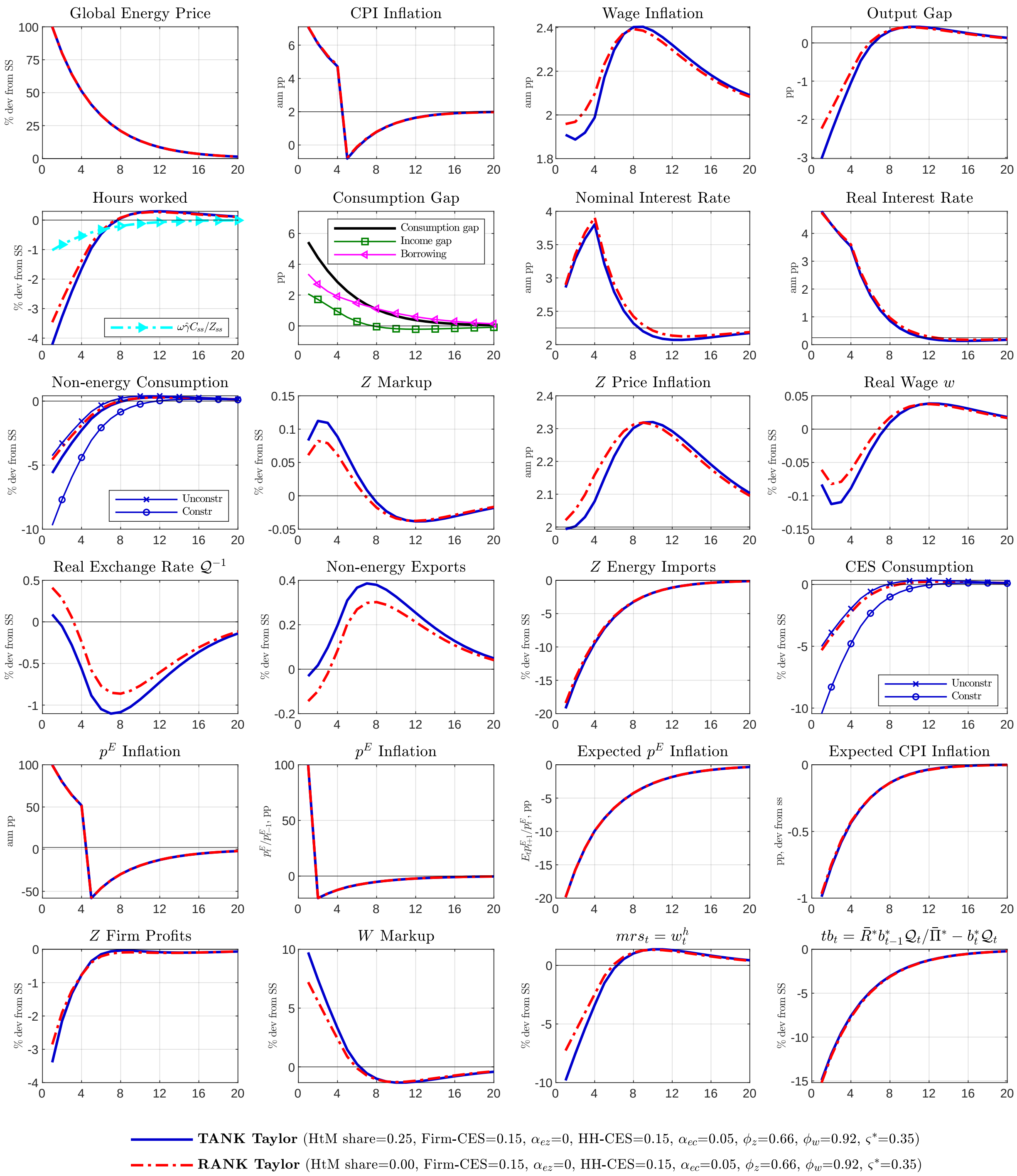


RANK (HtM share=0, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)
TANK a (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)
TANK b (HtM share=0.5, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of variables that pin down the policy stance ($C_{u,t}^{-1}$) to a 100 % increase in the foreign currency price of energy. In the column on the left (right) the central bank implements Ramsey-optimal policy (follows a Taylor rule). Energy enters only in the firm's production function. The red lines depict the RANK case. The blue crossed (circled) line depicts the TANK case *a* (*b*) with $\omega = 0.25$ ($\omega = 0.5$).

B.4 Energy shock with energy only in consumption, Taylor

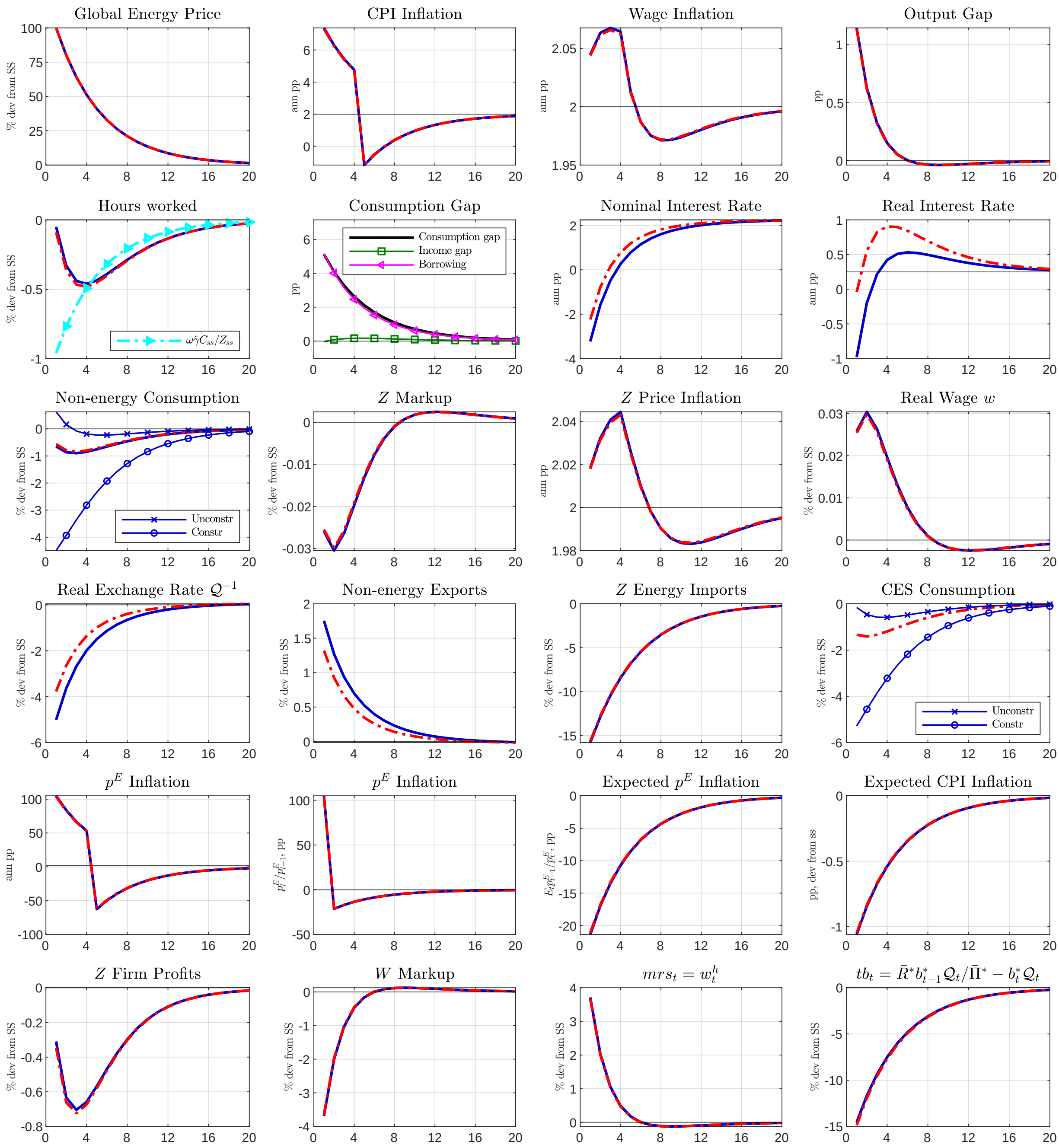
FIGURE B.5: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the consumption basket.

B.5 Energy shock with energy only in consumption, Ramsey

FIGURE B.6: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



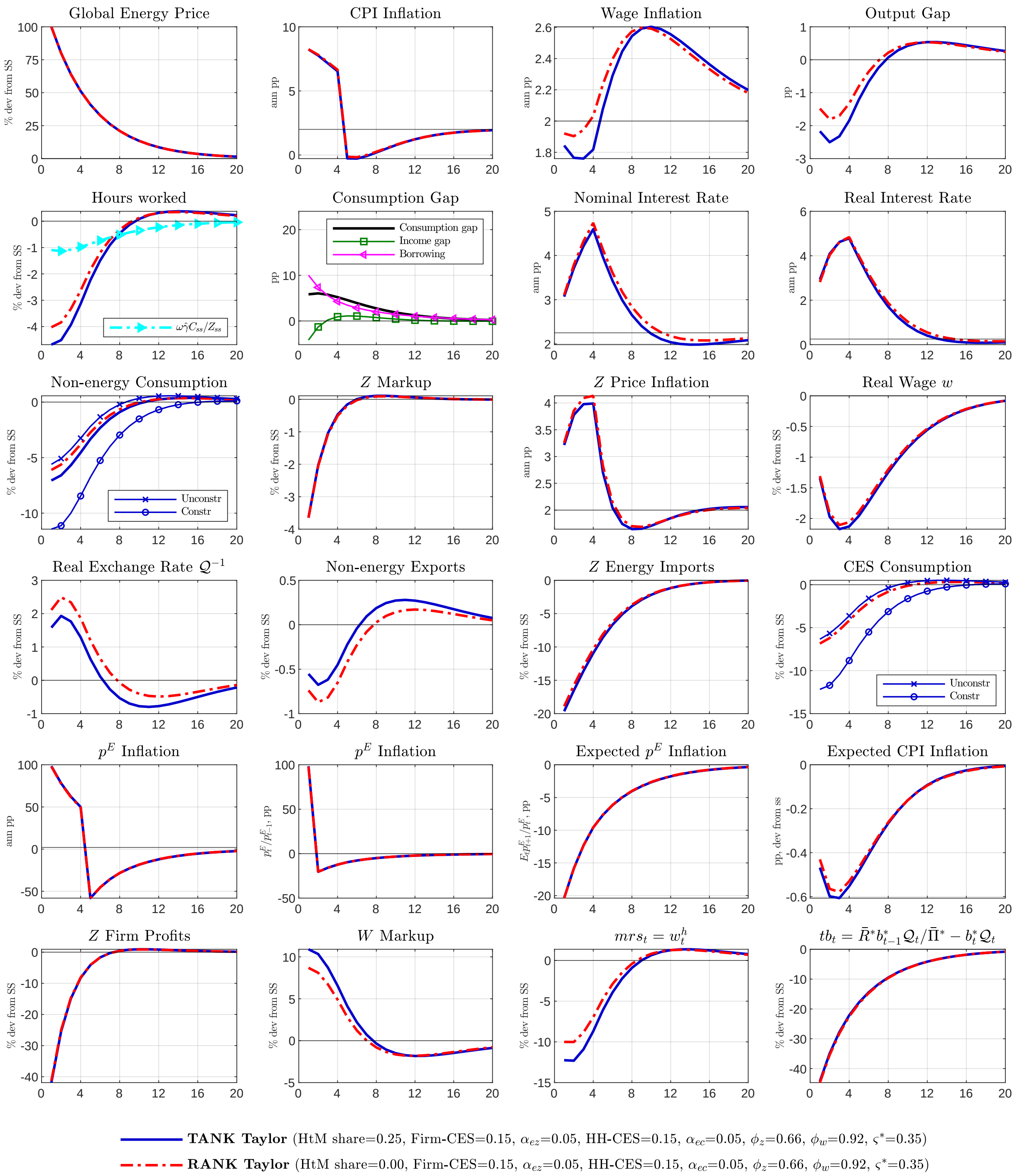
TANK Ramsey (HtM share=0.25, Firm-CES=0.15, α_{ez}=0, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

RANK Ramsey (HtM share=0.00, Firm-CES=0.15, α_{ez}=0, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only the consumption basket.

B.6 Energy shock with energy in production and consumption, Taylor

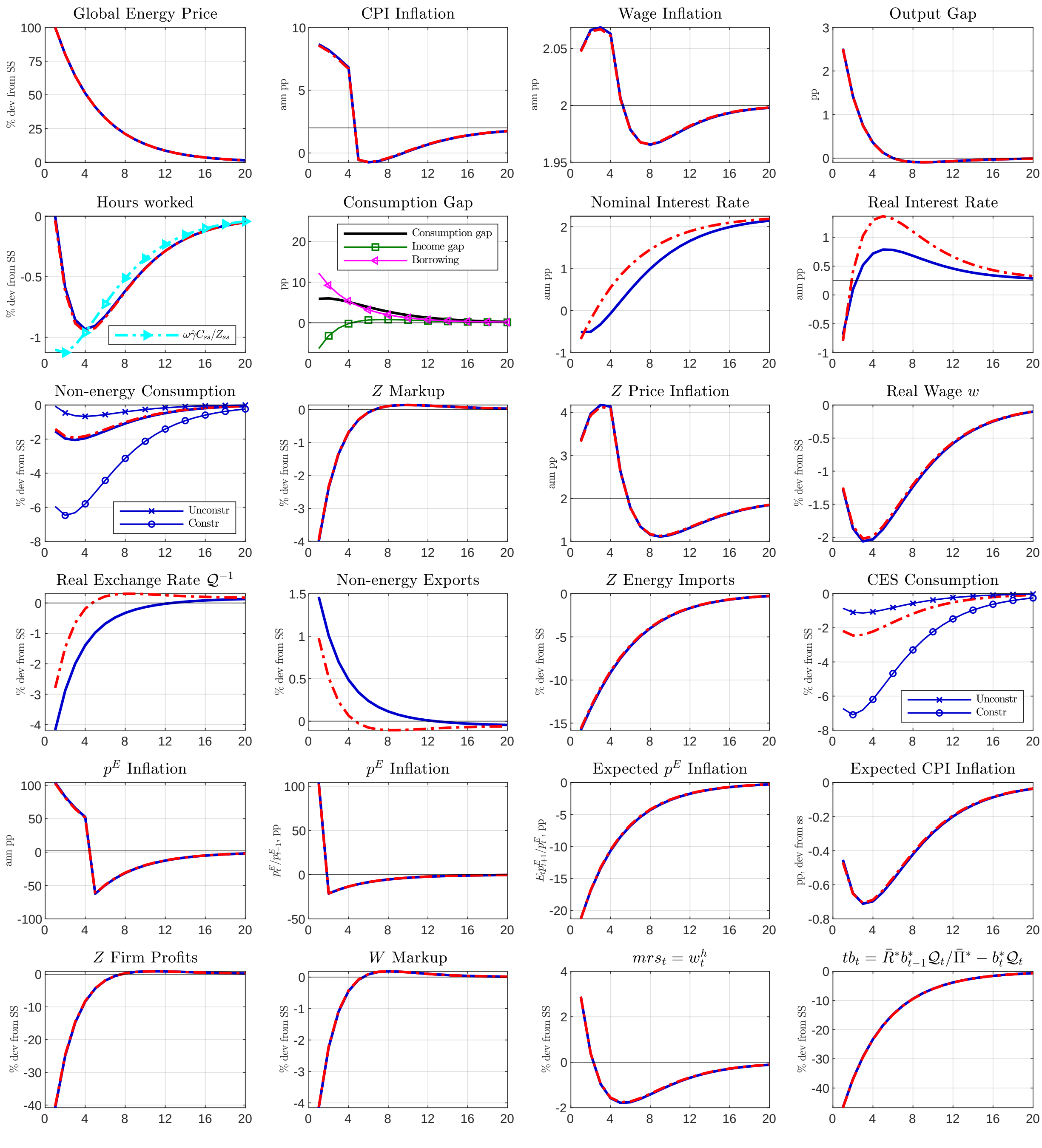
FIGURE B.7: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters the consumption basket and the production function.

B.7 Energy shock with energy in production and consumption, Ramsey

FIGURE B.8: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



TANK Ramsey (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

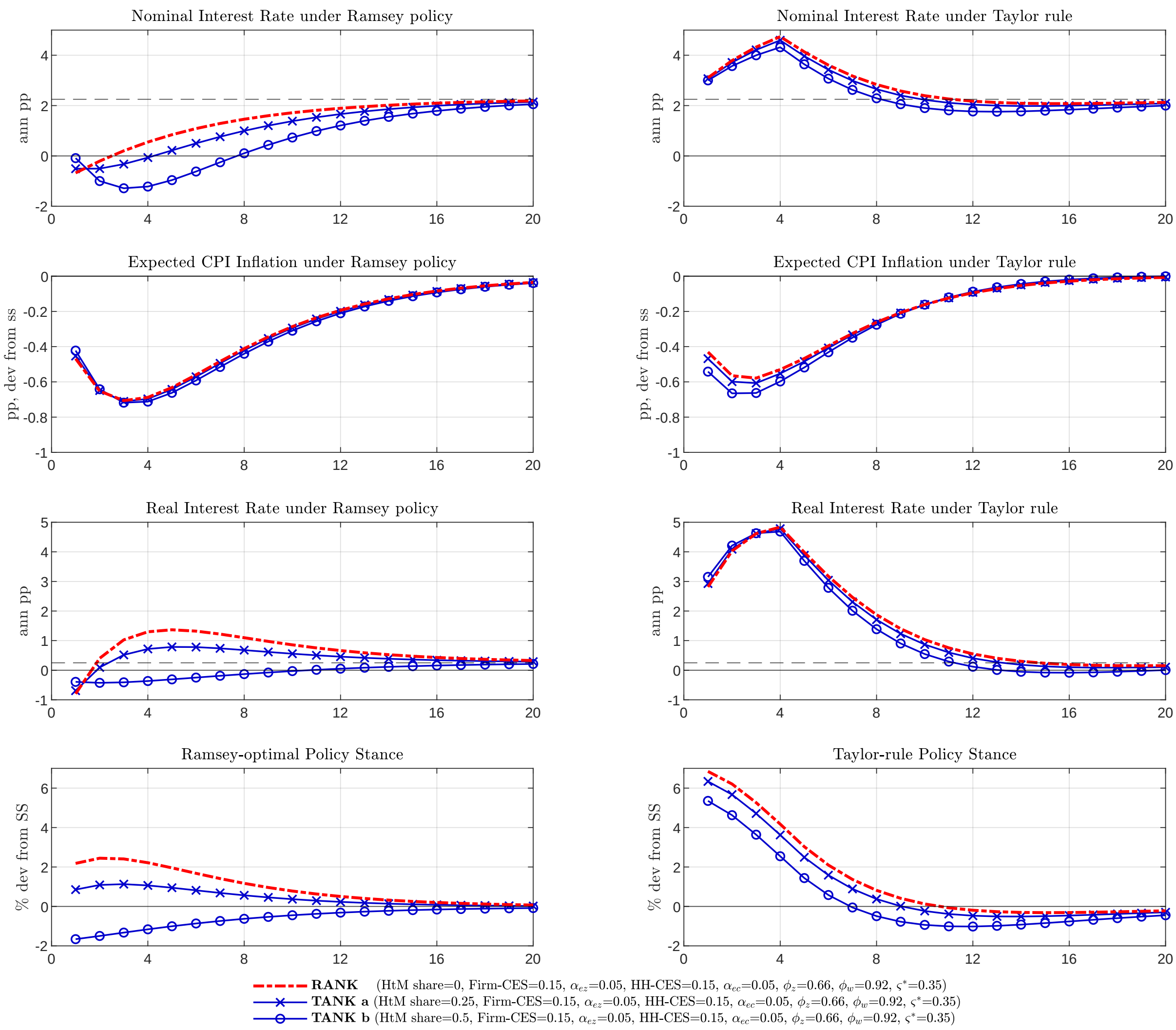
RANK Ramsey (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters the consumption basket and the production function.

B.8 Ramsey vs Taylor Policy Stance, Energy shock with energy in consumption and production

We compare the policy paths implied by a Taylor rule and implied by Ramsey-optimal policy in the case in which energy enters the production function and the consumption basket. Just as before in Figure 9, Figure B.9 presents the IRFs for the nominal interest rate (top row), the expected CPI inflation rate (second row from the top), the real interest rate (third row) and the policy stance (bottom row) over an increasingly larger share of constrained agents, which allows the energy price shock to yield a correspondingly larger fall in households' consumption. The left column in the chart depicts the case of Ramsey-optimal policy, the right column depicts the case of a Taylor rule.

FIGURE B.9: Dynamic Responses to a Global Energy Price Shock: Policy for Energy in Consumption and Production



Notes: This figure shows the IRFs of variables that pin down the policy stance ($C_{u,t}^{-1}$) to a 100 % increase in the foreign currency price of energy. In the column on the left (right) the central bank implements Ramsey-optimal policy (follows a Taylor rule). Energy enters in the consumption basket and in the firm's production function. The red lines depict the RANK case. The blue crossed (circled) line depicts the TANK case *a* (*b*) with a hand-to-mouth share of $\omega = 0.25$ ($\omega = 0.5$). Full set in Figure B.7 and B.8.

It can be seen in the upper left panel of Figure B.9 that the Ramsey planner cuts the nominal rate on impact and gradually lifts it back towards the steady state level of 2.25%. The fall in the nominal rate is so strong that the zero lower bound would be reached. Under a Taylor rule, the nominal rate is increased toward rougly 4% after 4 quarters, before falling back towards steady state.

The Ramsey-policy implied profile for the nominal rate in the case in which energy enters consumption and production is in stark contrast to the behaviour of the nominal rate under Ramsey policy in the case in which energy enters only in production, which we showed in Figure 9.

If energy also enters in the consumption basket, and if the price of energy jumps up on impact before falling back gradually, this implies that expected CPI inflation will turn negative, since expected energy price inflation turns negative. This can be observed in the second row of Figure B.9.

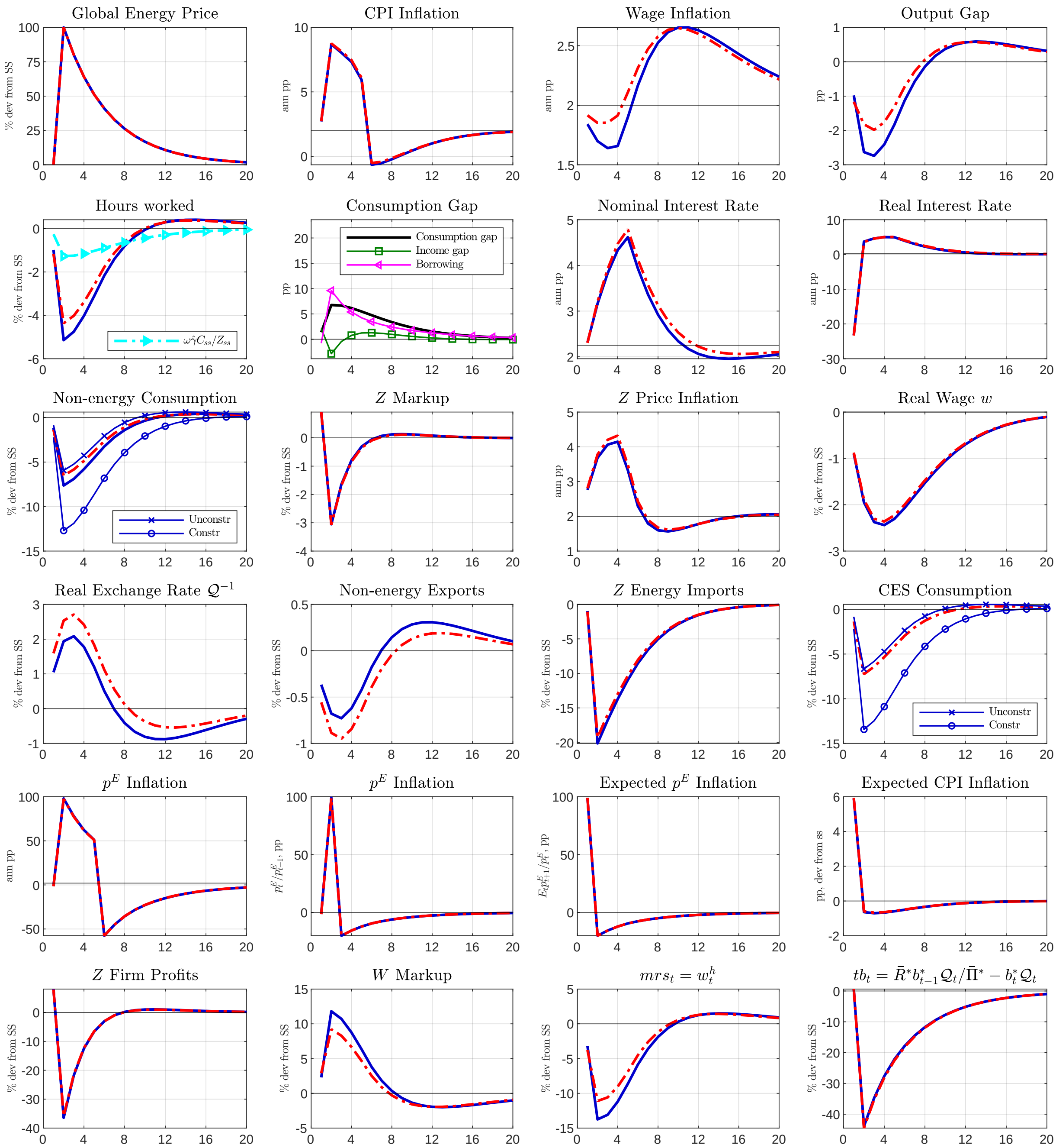
The real rate under a Taylor-rule would increase strongly, in part because the nominal rate is lifted, and in part because expected CPI inflation falls. The Ramsey planner implements a less contractionary stance by cutting the nominal rate. The lower left panel shows that for a Ramsey planner in the RANK model and in a TANK model with a moderate degree of credit constraints ($\omega = 0.25$) the policy stance is still contractionary, since the expected cumulative real rate path is positive. If the degree of credit constraints increases to $\omega = 0.5$, the policy stance becomes accommodative. In this regard, the optimal policy stance for the case in which energy enters consumption and production resembles the optimal policy stance in which energy only enters production. In both cases, in the case of energy only in production and in the case of energy in consumption and production, the Taylor-rule implied policy stance was too tight. Moreover, in both cases, the stance is less tight for a TANK model with strong credit constraints. However, the corresponding Ramsey-optimal paths for the nominal rate are very different, depending on whether expected CPI inflation falls or increases, in response to the energy price shock.

In a model with a delayed pass-through of energy import prices to domestic prices, it is likely that expected CPI inflation would not fall as much on impact as is the case in our model. In such a model with slower pass-through a Ramsey-planner would not optimally cut the nominal rate on impact.

B.9 Energy news shock with energy in production and consumption, Taylor

The normative implication of energy price shocks for monetary policy depend on whether the increase in energy prices is purely unanticipated. In Figure B.10 and B.11 we show the effects of an anticipated energy price 'news' shock. In this case, expected CPI inflation increases and the nominal rate under the Ramsey-optimal policy would need to increase sharply as well.

FIGURE B.10: Dynamic Responses to a Global Energy Price News Shock: Taylor policy for TANK vs RANK



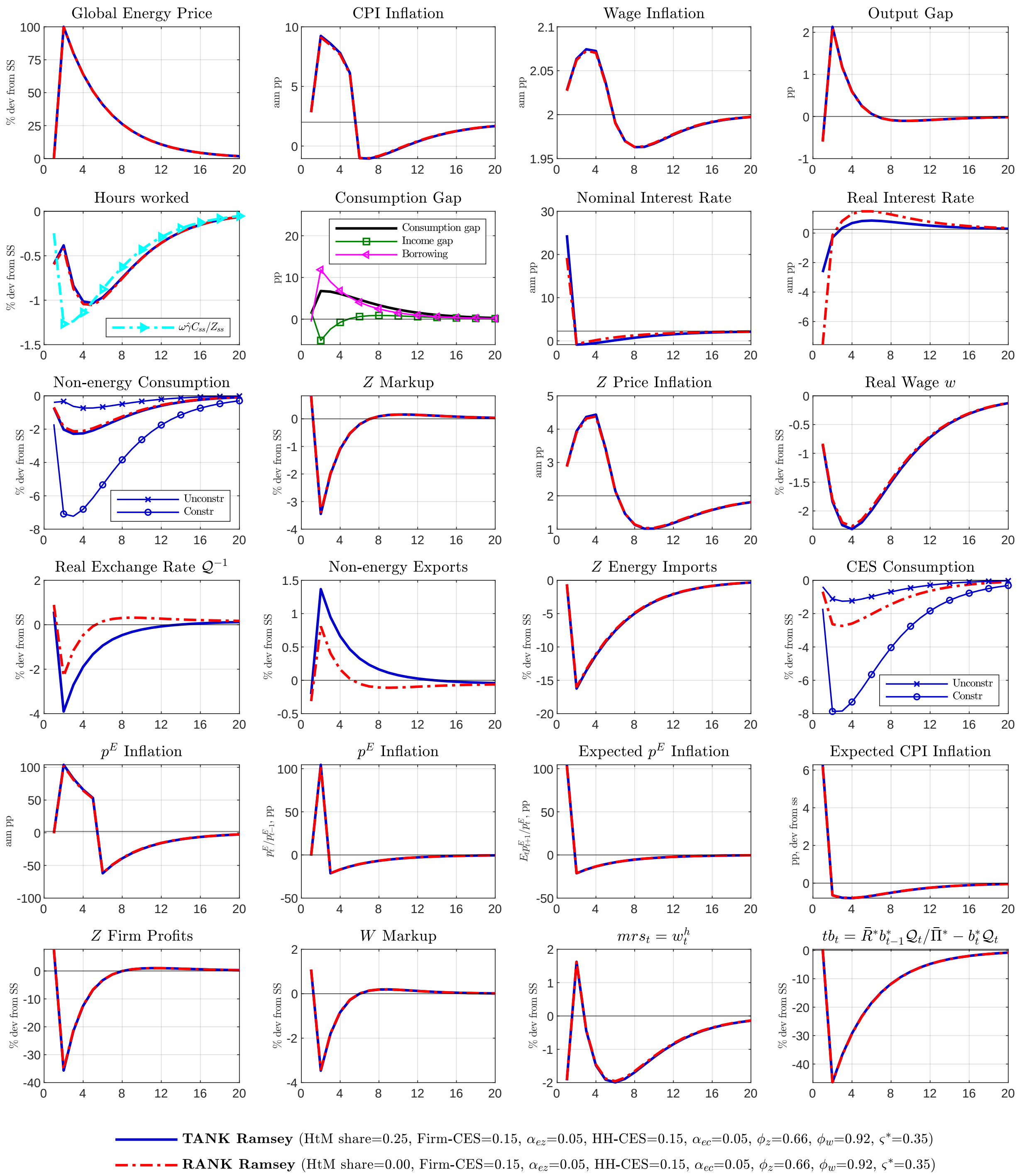
TANK Taylor (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

RANK Taylor (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0.05, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% anticipated increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters the consumption basket and the production function.

B.10 Energy news shock with energy in production and consumption, Ramsey

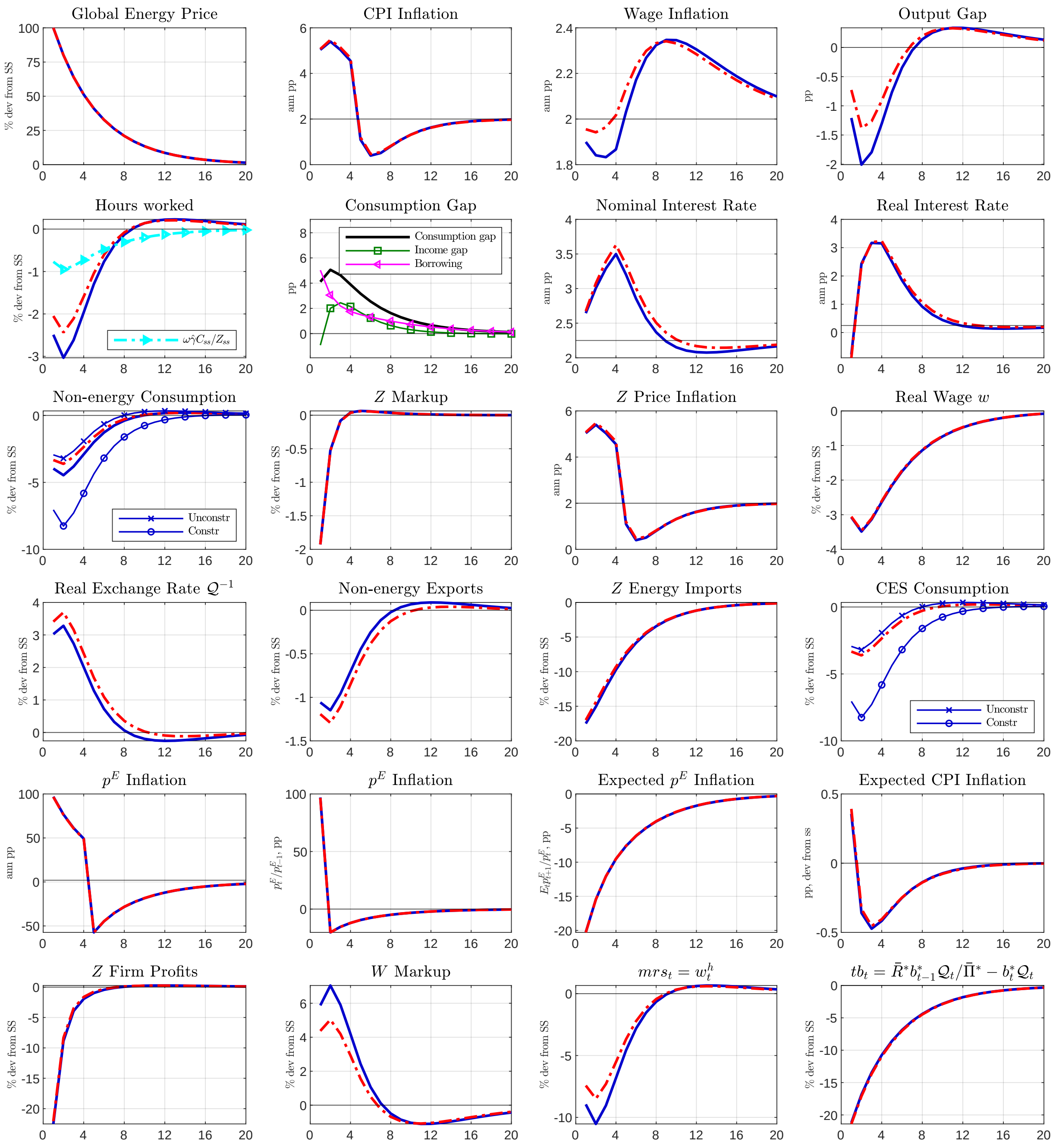
FIGURE B.11: Dynamic Responses to a Global Energy Price News Shock: Ramsey policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% anticipated increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters the consumption basket and the production function.

B.11 Energy shock under higher price flexibility, Taylor

FIGURE B.12: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



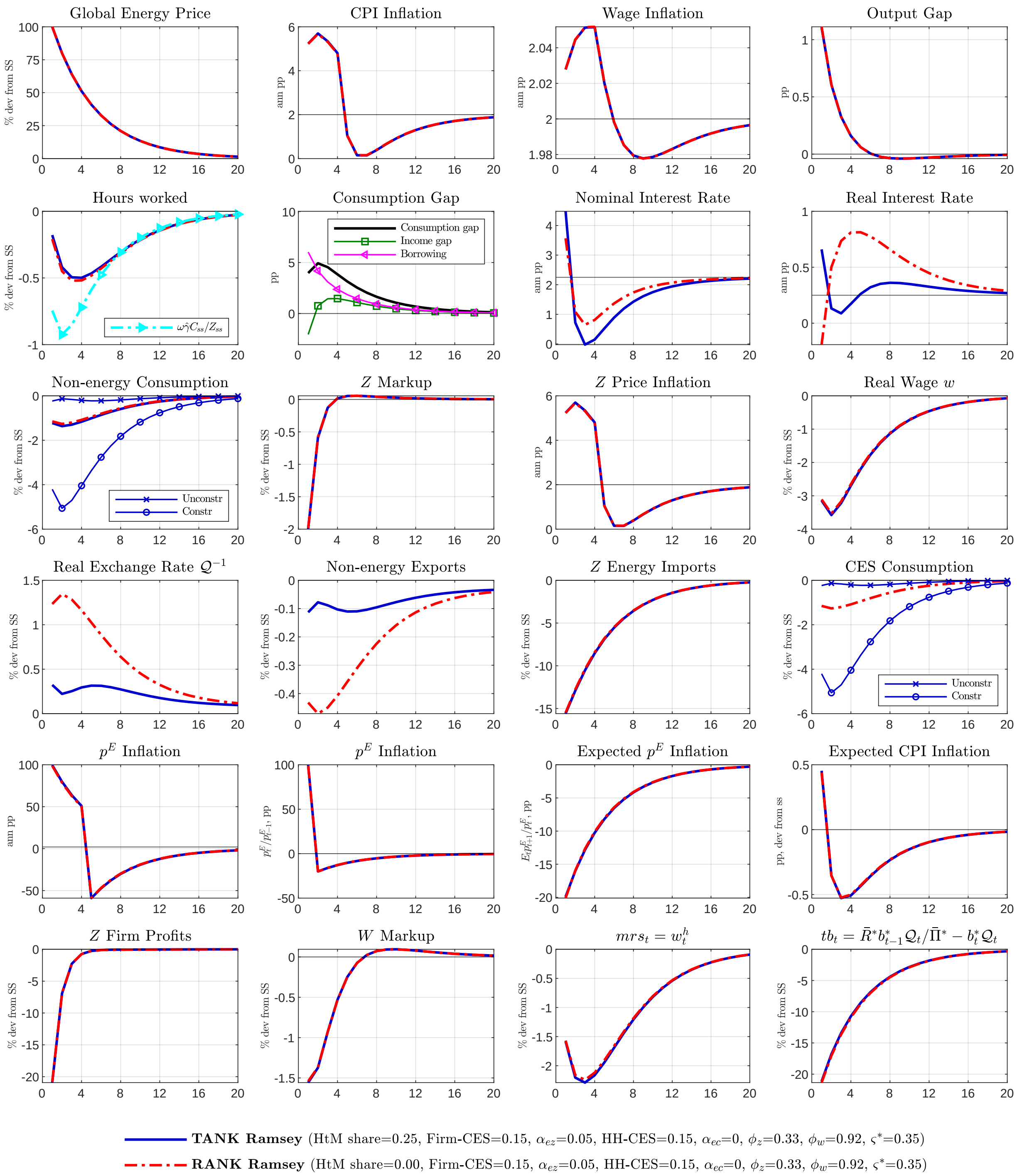
TANK Taylor (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.33, ϕ_w=0.92, ς^*=0.35)

RANK Taylor (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.33, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the production function.

B.12 Energy shock under higher price flexibility, Ramsey

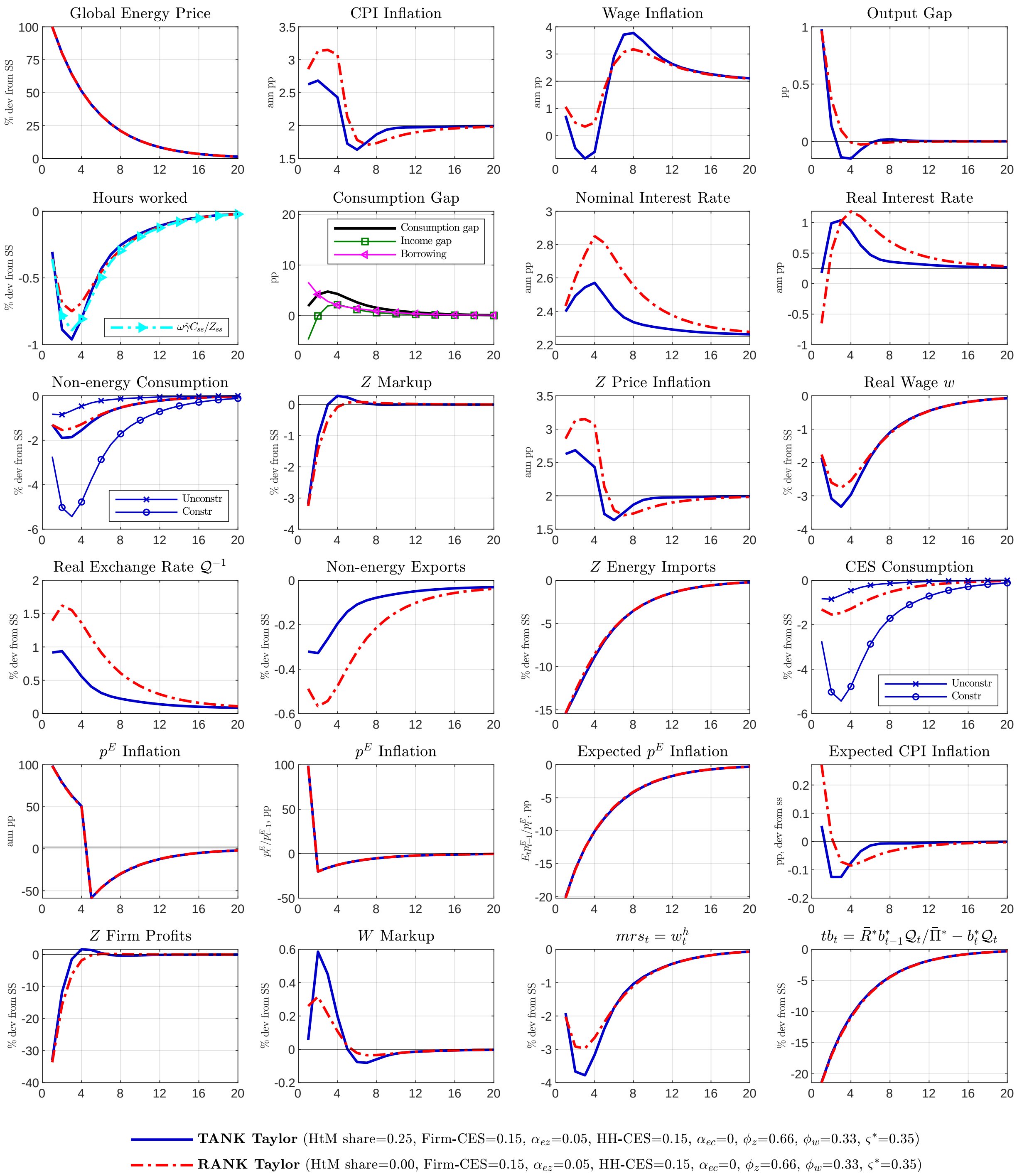
FIGURE B.13: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.

B.13 Energy shock under higher wage flexibility, Taylor

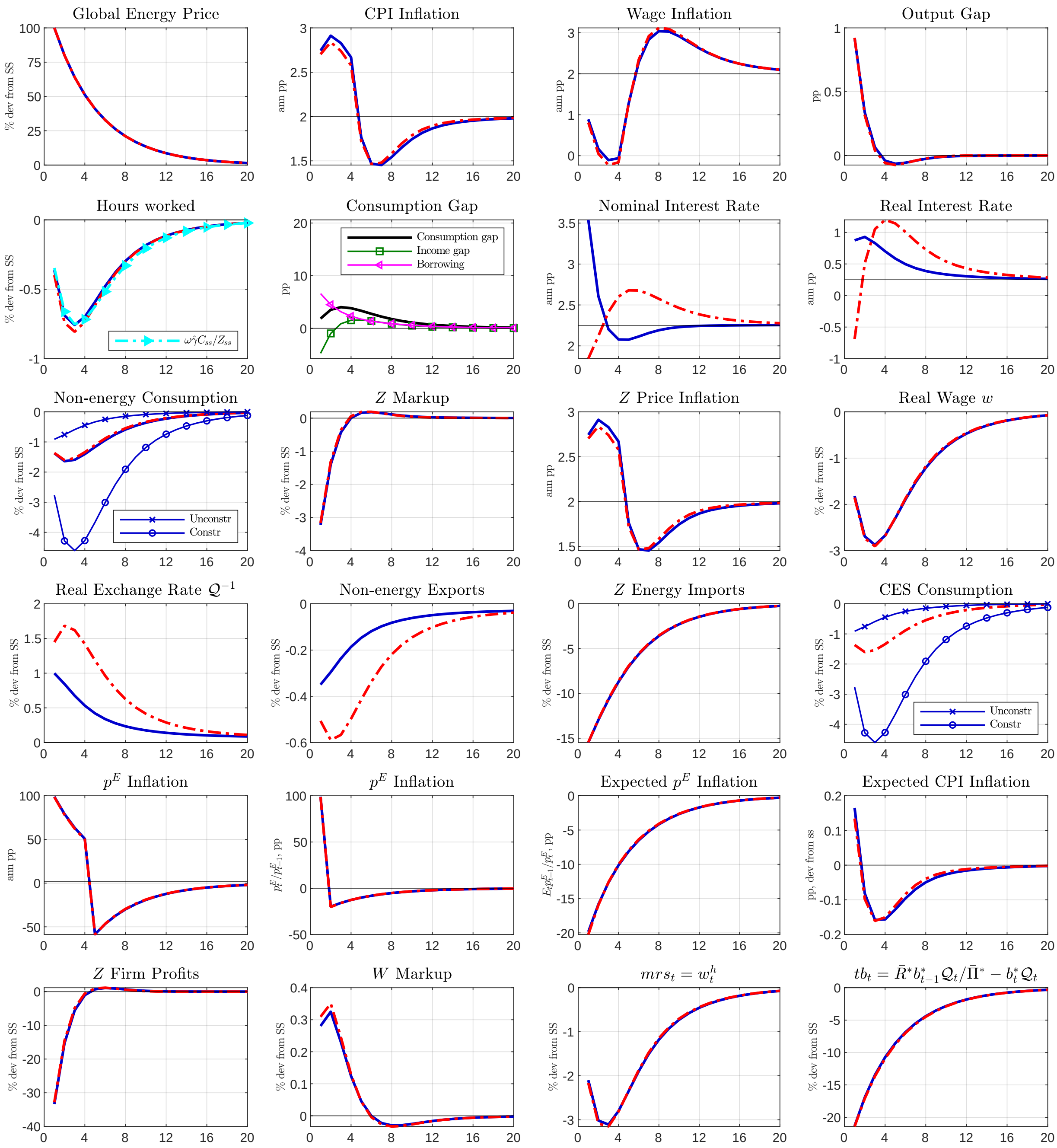
FIGURE B.14: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the production function.

B.14 Energy shock under higher wage flexibility, Ramsey

FIGURE B.15: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



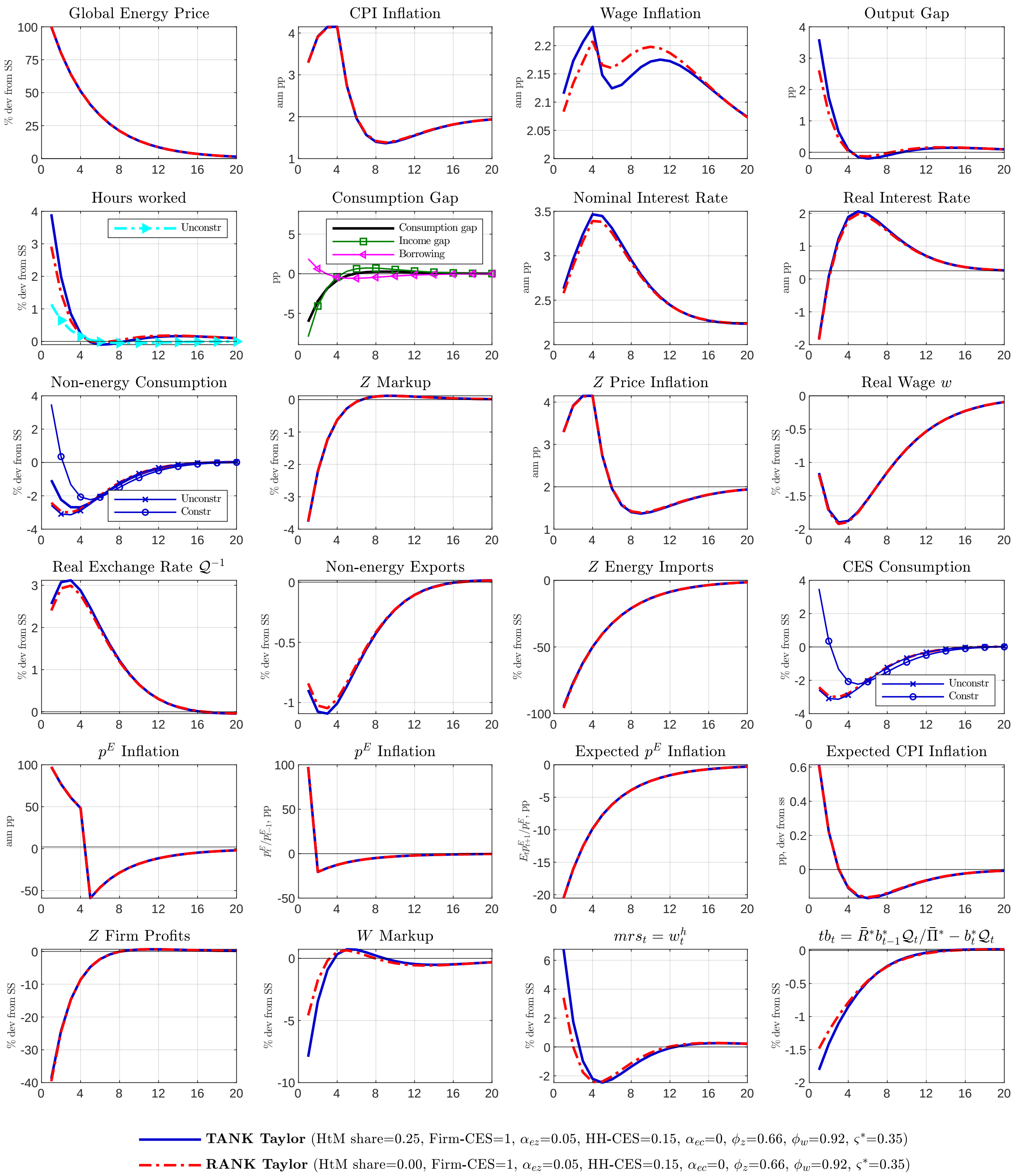
TANK Ramsey (HtM share=0.25, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.33, ς^*=0.35)

RANK Ramsey (HtM share=0.00, Firm-CES=0.15, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.33, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.

B.15 Energy shock under higher factor substitutability, Taylor

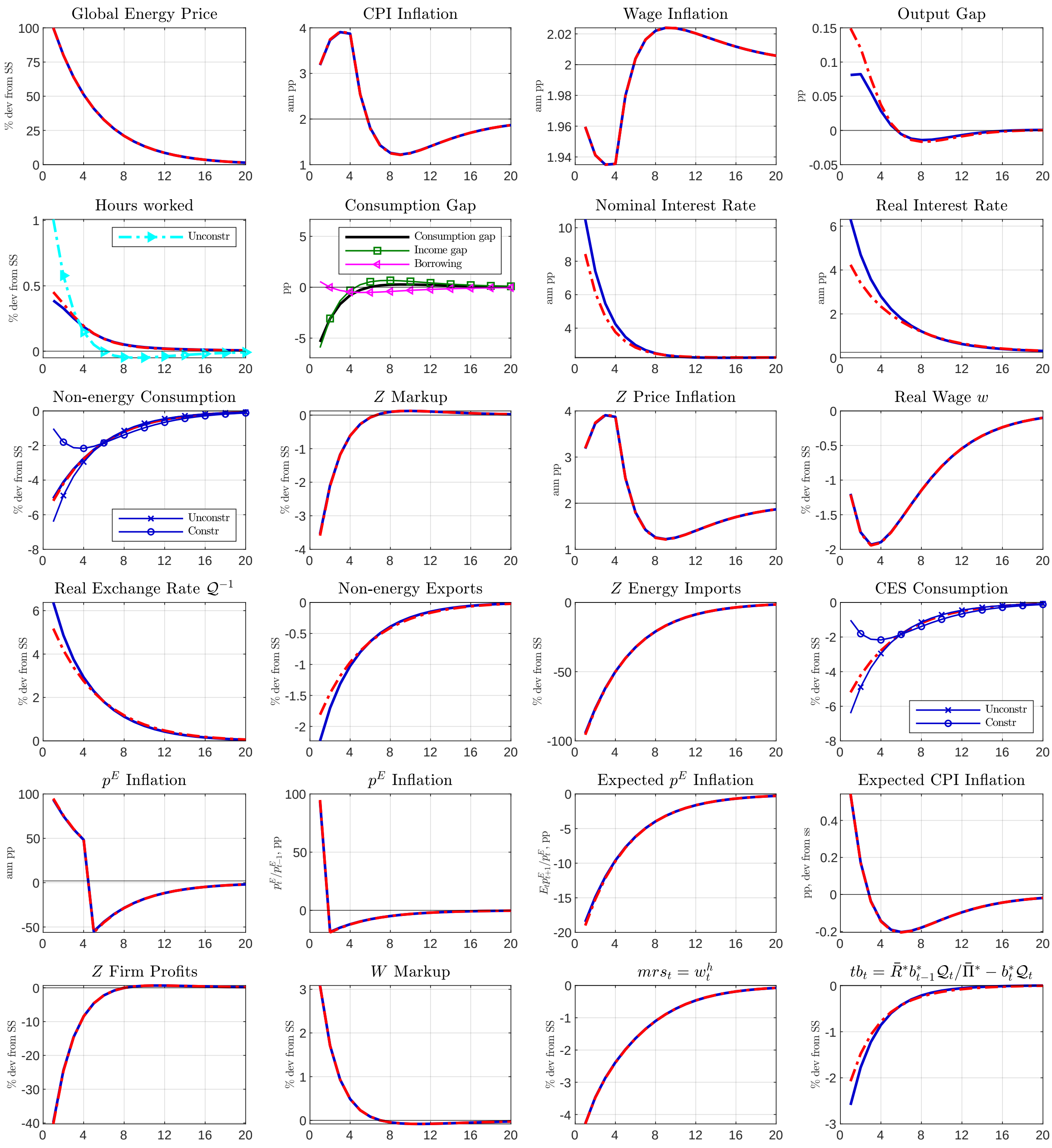
FIGURE B.16: Dynamic Responses to a Global Energy Price Shock: Taylor policy for TANK vs RANK



Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank follows a Taylor rule. The blue (red) lines depict the TANK (RANK) case. Energy enters only the production function.

B.16 Energy shock under higher factor substitutability, Ramsey

FIGURE B.17: Dynamic Responses to a Global Energy Price Shock: Ramsey policy for TANK vs RANK



TANK Ramsey (HtM share=0.25, Firm-CES=1, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

RANK Ramsey (HtM share=0.00, Firm-CES=1, α_{ez}=0.05, HH-CES=0.15, α_{ec}=0, ϕ_z=0.66, ϕ_w=0.92, ς^*=0.35)

Notes: This figure shows the IRFs of key model variables to a 100% increase in the foreign currency price of energy. The central bank implements Ramsey-optimal policy. The blue (red) lines depict the TANK (RANK) case. Energy enters only in the firm's production function.