# A Two-Agent New Keynesian (TANK) Model

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# Outline

# Motivation

- How do aggregate fluctuations arise, and how does monetary policy transmit to the real economy?
- Growing literature: re-examining "old" questions (e.g. monetary policy, fiscal transfers, stabilization) through models with heterogeneous households.
- Why focus on household heterogeneity?
  - Household heterogeneity is pervasive in data (income, wealth, borrowing constraints).
  - Aggregate dynamics can depend on the distribution of resources, not just averages.
  - Policies that are neutral in representative agent models often have large effects in heterogeneous agent settings.

# Outline

# Representative Agent New Keynesian Model (RANK)

- Households are identical
- Resource constraint

$$c_t = y_t$$

- Euler equation/IS curve pins down consumption:

$$c_t = \mathbb{E}_t c_{t+1} - \sigma \left( i_t - \mathbb{E}_t \pi_{t+1} - \tilde{r}_t \right)$$

- Effects of monetary policy:
  - **Direct/intertemporal substitution/partial equilibrium effect**: lower real rates $\rightarrow$ increase in current consumption.
  - **Indirect/income/general equilibrium effect**: higher demand $\rightarrow$ higher income and inflation.

# Representative Agent New Keynesian Model (RANK)

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Regardless of the labels, in the standard New Keynesian Model it is the **direct effect that does most of the work (95% in standard calibrations)**. As Bilbiie (2019) puts it, the general equilibrium New Keynesian Model is not very general equilibrium. Nor is it very Keynesian...

# Limits of RANK

- Intertemporal substitution (direct) effect dominates in RANK.
- But data suggest otherwise:
  1. Weak sensitivity of consumption to real interest rates (Hall, 1978; Campbell & Mankiw, 1989)
  2. Strong sensitivity of consumption to income (Jappelli & Pistaferri, 2010; Parker, Souleles, Johnson & McClelland, 2013)
  3. Transfers matter, violating Ricardian equivalence (Barro, 1974; Mankiw & Shapiro, 1985; Johnson, Parker & Souleles, 2006; Fagereng, Holm & Natvik, 2021)
  4. Household heterogeneity is substantial (Kaplan & Violante, 2014; Krueger, Mitman & Perri, 2016; Auclert, Rognlie & Straub, 2018)

$\Rightarrow$ RANK fails to capture key empirical features.

# Outline

# Motivation for TANK

- Remedy the “low MPC” in RANK.
- Two household types:
  - **Savers (S)**: unconstrained, respond to interest rates.
  - **Hand-to-mouth (H)**: constrained, respond only to current income.

$\rightarrow$ Hand-to-mouth consumers make consumption more responsive to income.

# How TANK Works

- Constrained agents face binding borrowing constraints.
- Their consumption tracks income, not interest rates.
- Average consumption = composite of S and H households $\rightarrow$ consumption gap responds to shocks.
- TANK leads to richer aggregate dynamics than RANK, while preserving tractability.

# Channels of Heterogeneity

Three channels where heterogeneity matters:

- **Gap**: difference in consumption between constrained and unconstrained households.
- **Dispersion**: distributional changes among unconstrained households.
- **Share**: fraction of constrained households in the population.

**RANK**: abstracts from all three.

**TANK**: captures the “gap” channel with tractability.

**HANK**: captures all three, less tractability.

# HANK vs TANK

**HANK:**

- Households face idiosyncratic shocks and incomplete markets.
- Endogenous, time-varying wealth distribution.
- Captures dispersion and share effects explicitly.

**TANK:**

- Abstracts from idiosyncratic shocks.
- Captures the "gap" channel in a simple way.
- Equilibrium conditions reduce to a system of equations isomorphic to a RANK
- Approximates HANK if share of constrained is well-calibrated.

When is TANK a suitable substitute for HANK? (Debortoli and Galí, 2025)

# Outline

# Notation

- Time is discrete: $t = 0, 1, 2, \ldots$
- Steady-state values carry no time subscript (e.g., $X$).
- **Real** quantities are in units of the consumption good and use lowercase (i.e., $c_t, w_t$).
- **Nominal** variables use uppercase (i.e., $R_t, \Pi_t$).
- Log-deviations from steady state use hats: $\hat{x}_t \equiv \log X_t - \log X$.
- Variables expressed as (linearized) shares of steady-state output use tildes: $\tilde{d}_t \equiv \frac{D_t - D}{Y}$, $\tilde{t}_t^H \equiv \frac{T_t^H - T^H}{Y}$.

# Economic Blocks

- **Households**: **Savers (S)** and **Hand-to-Mouth (H)**.
- **Firms**: standard NK structure with Rotemberg price stickiness.
- **Government**: redistributes via taxes/transfers.
- **Central Bank**: Taylor rule for the policy rate.

# Households: 2 Types

- Continuum of households on $[0,1]$ with common period utility $U(\cdot)$.
- Share $\lambda$ are **H** (hand-to-mouth): work and consume current income (plus transfers).
- Share $1-\lambda$ are **S** (savers): work, hold risk-free bonds and receive profits.

# Savers (S): Problem and Optimality

Savers maximize their lifetime utility subject to their budget constraint, given prices and wages

$$\max_{\{c_t^S, b_t^S, H_t^S\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^S)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^S \frac{(H_t^S)^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$c_t^S + b_t^S = \frac{1-\tau^D}{1-\lambda}\, d_t + w_t H_t^S + \frac{R_{t-1}}{\Pi_t} b_{t-1}^S.$$

FOCs:

$$(c_t^S)^{-1/\sigma} = \beta R_t\, \mathbb{E}_t \left[ \frac{(c_{t+1}^S)^{-1/\sigma}}{\Pi_{t+1}} \right], \qquad w_t = \nu^S (H_t^S)^{\varphi} (c_t^S)^{1/\sigma}.$$

Parameters: $\sigma$ (IES); $1/\varphi$ (Frisch elasticity); $\nu^S$ (leisure weight).

# Hand-to-Mouth (H): Problem and Optimality

HtM have no assets and thus consume their labor income as well as the transfer they get from the government:

$$\max_{\{c_t^H, H_t^H\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(c_t^H)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \nu^H \frac{(H_t^H)^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$c_t^H = w_t H_t^H + T_t^H.$$

FOC (labor supply):

$$w_t = \nu^H (H_t^H)^{\varphi} (c_t^H)^{1/\sigma}.$$

Parameters: $\sigma$ (IES); $1/\varphi$ (Frisch elasticity); $\nu^H$ (leisure weight).

# Firms: Technology and Price Setting

- **Final good**: CES aggregator over a continuum of differentiated intermediates.
- **Intermediate firms** (monopolistic competition):
  - Linear technology with productivity.
  - Downward-sloping demand $\Rightarrow$ markup power.
  - Rotemberg price adjustment costs $\propto \frac{\phi_p}{2}(\Pi_t - 1)^2 y_t$.
- **Key equilibrium conditions:**

$$w_t = mc_t \frac{y_t}{H_t},$$

$$y_t = z_t H_t,$$

$$0 = (1+\tau^S)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p (\Pi_t - 1) + \beta \mathbb{E}_t \left[ \left( \frac{c^S_{t+1}}{c^S_t} \right)^{-1/\sigma} \Pi_{t+1} \phi_p (\Pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \right].$$

# Policy: Fiscal Transfers and Taylor Rule

- **Redistribution (balanced each period)**:

$$\lambda\, t_t^H = \tau^D\, d_t .$$

- **Production subsidy**: chosen to induce marginal-cost pricing in steady state; financed by a lump-sum tax on firms

$$t_t^F = \tau^S y_t .$$

- **Monetary policy**: simple Taylor rule

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} e^{\epsilon_t^m} .$$

▸ Relative to Bilbiie (2008, 2019)

# Market Clearing and Resource Constraint

- Bond market clearing: $b_t^S = 0$.
- Aggregation:

$$c_t = \lambda c_t^H + (1-\lambda) c_t^S, \qquad H_t = \lambda H_t^H + (1-\lambda) H_t^S.$$

- Profits (under Rotemberg costs):

$$d_t = \left(1 - mc_t - \tfrac{\phi_p}{2}(\Pi_t - 1)^2\right) y_t.$$

- Resource constraint:

$$c_t = y_t - \tfrac{\phi_p}{2}(\Pi_t - 1)^2 y_t.$$

# Equilibrium Conditions: Summary

| | Equilibrium Conditions | |
|---|---|---|
| 1 | Labor supply (S) | $w_t = \nu^S (H_t^S)^{\varphi} (c_t^S)^{1/\sigma}$ |
| 2 | Labor supply (H) | $w_t = \nu^H (H_t^H)^{\varphi} (c_t^H)^{1/\sigma}$ |
| 3 | Euler (S) | $1 = \beta\, \mathbb{E}_t \left[ \left( \frac{c_{t+1}^S}{c_t^S} \right)^{-1/\sigma} \frac{R_t}{\Pi_{t+1}} \right]$ |
| 4 | Budget (H) | $c_t^H = w_t H_t^H + t_t^H$ |
| 5 | Transfers | $t_t^H = \frac{\tau^D}{\lambda} d_t$ |
| 6 | MPL | $w_t = mc_t \frac{y_t}{H_t}$ |
| 7 | Price setting | $[l](1+\tau^S)(1-\epsilon) + \epsilon mc_t - \Pi_t \phi_p (\Pi_t - 1)$ |
| | | $+ \beta \mathbb{E}_t \Big[ \left( \frac{c_{t+1}^S}{c_t^S} \right)^{-1/\sigma} \Pi_{t+1} \phi_p (\Pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \Big] = 0$ |
| 8 | Production | $y_t = z_t H_t$ |
| 9 | Profits | $d_t = \left( 1 - mc_t - \frac{\phi_p}{2} (\Pi_t - 1)^2 \right) y_t$ |
| 10 | Agg. consumption | $c_t = \lambda c_t^H + (1-\lambda) c_t^S$ |
| 11 | Agg. hours | $H_t = \lambda H_t^H + (1-\lambda) H_t^S$ |
| 12 | Resource constraint | $c_t = y_t - \frac{\phi_p}{2} (\Pi_t - 1)^2 y_t$ |
| 13 | Taylor rule | $\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\phi_\pi} e^{\epsilon_t^m}$ |

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| | | $+\beta \mathbb{E}_t \left[ \left( \frac{c_{t+1}^S}{c_t^S} \right)^{-1/\sigma} \Pi_{t+1} \phi_p (\Pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \right] = 0$ |
| 8 | Production | $y_t = z_t H_t$ |
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# Equilibrium Conditions: Log-linearized

| | | Log-linearized Conditions |
|---|---|---|
| 1 | Labor S | $\varphi \hat{H}_t^S = \hat{w}_t - \sigma^{-1}\hat{c}_t^S$ |
| 2 | Labor H | $\varphi \hat{H}_t^H = \hat{w}_t - \sigma^{-1}\hat{c}_t^H$ |
| 3 | Euler S | $\hat{c}_t^S = \mathbb{E}_t \hat{c}_{t+1}^S - \sigma(\hat{R}_t - \mathbb{E}_t \hat{\Pi}_{t+1})$ |
| 4 | Budget H | $\hat{c}_t^H = \hat{H}_t^H + \hat{w}_t + \tilde{t}_t^H$ |
| 5 | Transfers | $\tilde{t}_t^H = \frac{\tau^D}{\lambda}\tilde{d}_t$ |
| 6 | MPL | $\hat{w}_t = \hat{mc}_t + \hat{y}_t - \hat{H}_t$ |
| 7 | NKPC (Rotemberg) | $\hat{\Pi}_t = \beta \mathbb{E}_t \hat{\Pi}_{t+1} + \frac{\epsilon}{\phi_p}\hat{mc}_t$ |
| 8 | Production | $\hat{y}_t = \hat{z}_t + \hat{H}_t$ |
| 9 | Profits | $\tilde{d}_t = -\hat{mc}_t$ |
| 10 | Agg. C | $\hat{c}_t = \lambda \hat{c}_t^H + (1-\lambda)\hat{c}_t^S$ |
| 11 | Agg. H | $\hat{H}_t = \lambda \hat{H}_t^H + (1-\lambda)\hat{H}_t^S$ |
| 12 | Resource | $\hat{c}_t = \hat{y}_t$ |
| 13 | Taylor | $\hat{R}_t = \phi_\pi \hat{\Pi}_t + \epsilon_t^m$ |

Variables with $\hat{\cdot}$ are log-deviations from steady state; profits $\tilde{d}_t$ and transfers $\tilde{t}_t^H$ are linearized as shares of steady-state output.

# Outline

# TANK

- TANK is a tractable way to introduces heterogeneity into the New Keynesian framework.
- Key feature: a fraction $\lambda$ of households consume only out of current income.
- This changes how aggregate demand reacts to monetary and fiscal shocks.
- Roadmap:
  1. Simplify the model (no productivity shocks)
  2. Derive HtM consumption behavior and the elasticity $\chi$.
  3. Aggregate dynamics
  4. Inverted Aggregate Demand Logic (IADL)

## Step 1: Simplify the baseline model (no productivity shocks)

- With additivity and no steady-state consumption inequality, aggregate labor supply mirrors the individual one:

$$\varphi \hat{H}_t = \hat{w}_t - \sigma^{-1} \hat{c}_t.$$

- With $\hat{z}_t = 0$:

$$\hat{y}_t = \hat{c}_t = \hat{H}_t, \quad \hat{w}_t = \hat{mc}_t, \quad \tilde{d}_t = -\hat{w}_t.$$

- Therefore from aggregate labor supply:

$$\hat{w}_t = (\varphi + \sigma^{-1})\hat{c}_t. \qquad \textbf{(??)}$$

$\rightarrow$ wages and consumption co-move

# Reduced log-linear system (no productivity shocks)

| | | Log-linearized conditions |
|---|---|---|
| 1 | Labor S | $\varphi \hat{H}_t^S = \hat{w}_t - \sigma^{-1}\hat{c}_t^S$ |
| 2 | Labor H | $\varphi \hat{H}_t^H = \hat{w}_t - \sigma^{-1}\hat{c}_t^H$ |
| 3 | Euler S | $\hat{c}_t^S = E_t \hat{c}_{t+1}^S - \sigma(\hat{R}_t - E_t \hat{\Pi}_{t+1})$ |
| 4 | Budget H | $\hat{c}_t^H = \hat{H}_t^H + \hat{w}_t + \tilde{t}_t^H$ |
| 5 | Transfer H | $\tilde{t}_t^H = -\frac{\tau^D}{\lambda}\hat{w}_t$ |
| 6 | NKPC | $\hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \psi\, \hat{w}_t$ |
| 7 | Agg. C | $\hat{c}_t = \lambda \hat{c}_t^H + (1-\lambda)\hat{c}_t^S$ |
| 8 | Agg. H | $\hat{c}_t = \lambda \hat{H}_t^H + (1-\lambda)\hat{H}_t^S$ |
| 9 | Taylor | $\hat{R}_t = \phi_\pi \hat{\Pi}_t + \epsilon_t^m$ |

**Table 1:** Baseline TANK without productivity shocks

# Step 2: HtM consumption mechanics

HtM budget constraint and labor supply:

$$\hat{c}_t^H = \hat{w}_t + \hat{H}_t^H + \frac{\tau^D}{\lambda}\tilde{d}_t, \qquad \hat{H}_t^H = \frac{1}{\varphi}\hat{w}_t - \frac{1}{\varphi\sigma}\hat{c}_t^H.$$

Combine:

$$\hat{c}_t^H = \frac{\varphi + 1 - \varphi(\tau^D/\lambda)}{\varphi + \sigma^{-1}}\,\hat{w}_t.$$

Substitute aggregate wage condition and $c_t = y_t$:

$$\hat{c}_t^H = \chi\,\hat{y}_t, \quad \chi \equiv 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right).$$

$\rightarrow \chi$ summarizes how strongly HtM consumption co-moves with aggregate income, depending on the share of HtM ($\lambda$), redistribution ($\tau^D$) and labor supply elasticity ($\varphi$).

# Step 3: Savers' consumption and the Euler equation

- Aggregating:

$$\hat{c}_t^S = \frac{1-\lambda\chi}{1-\lambda}\,\hat{c}_t.$$

- Plug into the Euler equation of Savers $\Rightarrow$ aggregate Euler equation:

$$\hat{c}_t = E_t\hat{c}_{t+1} - \delta^{-1}\hat{r}_t, \quad \delta = \frac{1-\lambda\chi}{1-\lambda}\cdot\frac{1}{\sigma}.$$

- **Interpretation:**
  - When $\chi = 1$ (i.e., $\varphi = 0$, $\sigma = 1$, $\tau^D = 0$), then $\hat{c}_t^S = \hat{c}_t$ and TANK collapses to RANK.
  - When $\chi > 1$, $\delta^{-1}$ rises: monetary policy effects are amplified: demand is more sensitive to real interest rates than in RANK.
  - $\chi$ encompasses the compositional effects (HtM share, transfers, Frisch).

## Step 4: Inverted Aggregate Demand Logic (IADL)

- Aggregate demand elasticity to real rates is $\delta^{-1}$. Its sign flips when

$$\delta = 0 \quad \Longleftrightarrow \quad \lambda = \lambda^{\star} \equiv \frac{\tau^{D}\varphi + 1}{\varphi + 1}.$$

- Regions:
  - If $\lambda < \lambda^{\star}$: **Standard logic (SADL)**. Higher real rates *reduce* demand (standard NK).
  - If $\lambda > \lambda^{\star}$: **Inverted logic (IADL)**. Higher real rates *raise* demand (inverted).
- As $\lambda \to \lambda^{\star}$, $|\delta^{-1}|$ is large: policy is very potent. As $\lambda \to 1$, $\delta^{-1} \to 0$: policy loses traction when no one holds assets.
- **Intuition:** With many HtM households and wage-sensitive income, the wage channel dominates. Interest rate increases can raise wages and profits, which feeds back into higher demand.

## SADL/IADL region

For SADL to apply

- the Frisch elasticity of labor supply (and of intertemporal substitution in labor supply), should be high ($\varphi$ should be low)
- and the higher $\frac{1}{\varphi}$, the higher the share of hand-to-mouth ($\lambda$) can be.

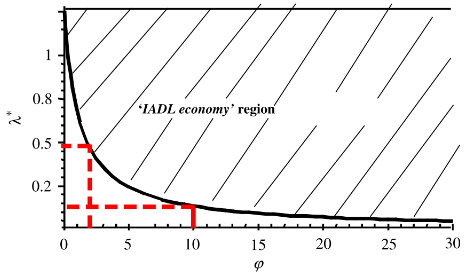


Fig. 1. Threshold share of non-asset holders as a function of inverse labor supply elasticity.

# Why IADL matters

- Shows how heterogeneity changes the impact of monetary policy
- Distribution (who holds assets, who earns wages) matters for aggregate dynamics.
- Even a simple TANK can deliver substantially different dynamics compared to RANK.

# TANK: Inverted Aggregate Demand Logic

- IADL case occurs when enough agents consume their wage income $w_t$ ($\lambda$ is high) and/or wage is sensitive enough to real income $y_t$ ($\varphi$ high). See section 3.1 in Bilbiie (2008) for an intuitive explanation.
- How can an increase in interest rates become expansionary when asset market participation is restricted enough?
  - Consider a one-time discretionary increase ($\epsilon_t^M$) in the interest rate, $\hat{R}_t = E_t \hat{\Pi}_{t+1} + \epsilon_t^M$.

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  - In RANK ($\lambda = 0$), aggregate demand falls. RA that hold assets is willing to work more at a given real wage (labor supply shifts rightward), but labor demand shifts left because of sticky prices (not all the fall in demand can be accommodated via cutting prices). The new equilibrium is one with lower output, consumption, hours and real wage.

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  - If $\lambda < \lambda^\star$ (SADL), the fall in real wage generated by the intertemporal substitution of savers now means a **further** fall in demand, since hand-to-mouth only consume their wage income. This generates a further shift in labor demand and output, consumption, hours and wages go down even further.

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  - **Why is this effect non-monotonic?**

# TANK: Inverted Aggregate Demand Logic - Additional Distributional Dimension

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  - Labor demand has to shift rightward; the equilibrium is reached where labor demand increases enough to generate an increase in the real wage, and enough not to generate a too strong fall in profits.
  - Equilibrium where $\uparrow c, y, H, w$ - **expansionary monetary contraction**.

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- This is the reason why we observe this non-monotonicity in the elasticity of aggregate demand to interest rate $\delta^{-1}$.
- In the standard region, an increase in the share of H2M implies that monetary policy is more effective at containing aggregate demand
- Whereas in the ‘inverted’ region, monetary policy contractions are less expansionary as the share of H2M increases.

# Outline

## Application: Energy price shocks in TANK

- How does heterogeneity affect the transmission of energy price shocks?
- Small open economy TANK model (Chan, Diz and Kanngiesser, 2024):
  - CES production with low substitutability between labor and imported energy.
  - Constrained HHs: consume only out of labor income.
  - Unconstrained HHs: earn wages, profits, and have access to credit/markets.
- **Question:** How do shocks redistribute resources between groups, and how does that affect aggregate demand and policy?

# Preview of key results

- For an energy-importing country, higher global energy prices = terms-of-trade shock.
- Complementarity between energy and labor reduces the labor share of total factor expenditures.
- Constrained HHs depend only on labor income $\Rightarrow$ their consumption falls sharply.
- Aggregate demand depends on how the cost of the energy price shock is distributed between constrained and unconstrained households.

# Model Ingredient 1: Household Heterogeneity

**Budget constraint of "constrained" (hand-to-mouth) HHs:**

$$\underbrace{W_t^h N_{c,t}^h}_{\text{Labour Income}} = \underbrace{P_t C_{c,t}}_{\text{Consumption}} + \underbrace{T_{c,t}}_{\text{Transfers}} \tag{1}$$

**Budget constraint of "unconstrained" HHs:** firm profits & access to credit

$$\underbrace{W_t^h N_{u,t}^h}_{\text{Labour Income}} + \underbrace{R_{t-1} B_{u,t-1} + \bar{R}^* B_{u,t-1}^* \mathcal{E}_t}_{\text{Return on Savings}} + \underbrace{DIV_{u,t}^F}_{\text{Firm Profits}} = \underbrace{P_t C_{u,t}}_{\text{Consumption}} + \underbrace{B_{u,t} + B_{u,t}^* \mathcal{E}_t}_{\text{Savings}} + \underbrace{T_{u,t}}_{\text{Transfers}}$$

A share of $\lambda$ households will be constrained; $(1-\lambda)$ will be unconstrained.

$$C_t = (1-\lambda) C_{u,t} + \lambda C_{c,t}, \quad \text{Consumption Gap: } \Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}}, \quad \hat{\gamma}_t \equiv \log(\Gamma_t / \Gamma_{ss})$$

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# Model Ingredient 2: CES production

**CES production for final output $Z$ uses energy imports $E^z$ and labour $N$ as inputs:**

$$Z_t(i) \quad = \quad \varepsilon_t^{TFP} \left( (1 - \alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(N_t(i)\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} + (\alpha_{ez})^{\frac{1}{\psi_{ez}}} \left(E_t^z(i)\right)^{\frac{\psi_{ez}-1}{\psi_{ez}}} \right)^{\frac{\psi_{ez}}{\psi_{ez}-1}} \tag{2}$$

where $\psi_{ez}$ is the elasticity of substitution between labor and energy.

**Remaining Model Features**

1. Nominal rigidities in final output production and wage setting.
2. No capital; no other real rigidities.
3. Monetary Policy follows Taylor Rule
4. Three shocks: (i) global energy price $p_t^{E,*}$, (ii) TFP $\varepsilon_t^{TFP}$, (iii) price markup $\varepsilon_t^{\mathcal{M}z}$.

# Demand-side Effects of Energy Price Shocks

IS equation for domestic value-added employment $\hat{n}_t$:

$$\hat{n}_t = \underbrace{-\frac{1}{\sigma}\frac{C_{ss}}{Z_{ss}}\mathbb{E}_t\sum_{k=0}^{\infty}(\hat{r}_{t+k}-\hat{\pi}_{t+k+1})}_{\text{Inter-temporal substitution (-)}} + \underbrace{\frac{X_{ss}}{Z_{ss}}\varsigma^*\hat{q}_t}_{\text{ToT effect (-)}} \underbrace{+\psi_{ez}\alpha_{ez}\left(\hat{p}^E_t-\hat{w}_t\right)}_{\text{Input substitution in production (+)}} \underbrace{-\lambda\frac{C_{ss}}{Z_{ss}}\hat{\gamma}_t}_{\text{Consumption gap (+/-)}}$$

Channels through which an energy price shock impacts activity:

- Real interest rates, ToT: standard supply shock transmission channel in **RANK**.
- **Relative price of energy** $\hat{p}^E_t$: higher energy prices shift firms' input mix.
- **Consumption gap** $\hat{\gamma}_t$: distributional channel, only in **TANK**.

  $\rightarrow$ we know: $\partial\hat{n}_t/\partial\hat{\gamma}_t < 0$ $\rightarrow$ decompose $\hat{\gamma}_t$ and check: $\partial\hat{\gamma}_t/\partial\hat{p}^E_t = ?$

What affects the allocation of resources between constrained and unconstrained HHs in response to an energy price shock?

# Consumption Gap Decomposition

- Decompose the consumption gap:

$$\Gamma_t \equiv \frac{C_{u,t}}{C_{c,t}} = \frac{INC_{u,t} - \mathcal{E}_t \Delta B^*_{u,t} + \mathcal{E}_t(\bar{R}^* - 1)B^*_{u,t-1}}{INC_{c,t}} = \underbrace{\Gamma^{inc}_t}_{\text{income gap}} - \underbrace{\frac{1}{1-\lambda}\frac{TB_t}{INC_{c,t}}}_{\text{borrowing}}, \tag{3}$$

where the 'income gap' is the ratio of HH's incomes $\Gamma^{inc}_t = INC_{u,t}/INC_{c,t}$ and $TB_t = P_t X_t - P^E_t E^z_t$ denotes the trade balance.

- Two components:
  1. Income gap: relative exposure to wages vs. profits.
  2. Borrowing gap: differences in access to credit/foreign borrowing.
- Next, rewrite equation (3) in terms of firm markups ($\mathcal{M}^Z_t$) and the labour share of total factor expenditure ($\Xi_t \equiv W_t N_t \; /(W_t N_t + P^E_t E^z_t)$).

## Consumption Gap: Role of Markup and Labor Share

$$\Gamma_t = 1 + \underbrace{\frac{1}{1-\lambda}\frac{\mathcal{M}_t^Z - 1}{\Xi_t}}_{\text{income gap}} + \underbrace{\frac{1}{1-\lambda}\left(\frac{1}{\Xi_t} - 1 - \frac{P_t X_t}{INC_{c,t}}\right)}_{\text{borrowing}}. \tag{4}$$

→ The consumption gap is increasing in **firm markups**, $\partial\Gamma_t/\partial\mathcal{M}_t^Z > 0$.

→ The consumption gap is decreasing in the **labour share**, $\partial\Gamma_t/\partial\Xi_t < 0$.

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→ The consumption gap is increasing in **firm markups**, $\partial \Gamma_t / \partial \mathcal{M}_t^Z > 0$.
→ The consumption gap is decreasing in the **labour share**, $\partial \Gamma_t / \partial \Xi_t < 0$.

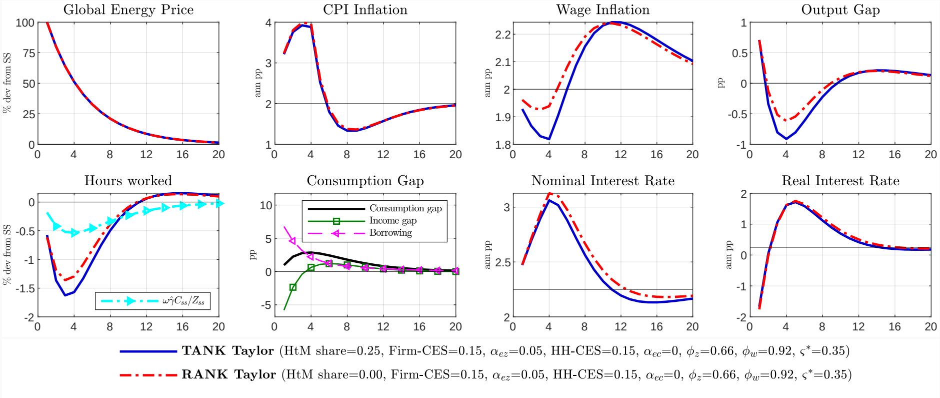
Next, decompose firms' markup and labor share:

$$\mathcal{M}_t^Z = \frac{\varepsilon_t^{TFP} P_t}{\left((1-\alpha_{ez}) W_t^{1-\psi_{ez}} + \alpha_{ez}\left(P_t^E\right)^{1-\psi_{ez}}\right)^{\frac{1}{1-\psi_{ez}}}},$$

$$\Xi_t = \left(1 + \frac{\alpha_{ez}}{1-\alpha_{ez}}\left(\frac{P_t^E}{W_t}\right)^{1-\psi_{ez}}\right)^{-1}.$$

→ Given price rigidities, an increase in $P_t^E$ reduces firms' markups ($\mathcal{M}_t^Z$), $\frac{\partial \mathcal{M}_t^Z}{\partial P_t^E} < 0$.
→ With low substitutability ($\psi_{ez} < 1$), an increase in $P_t^E$ reduces ($\Xi_t$), $\partial \Xi_t / \partial P_t^E < 0$.

**Figure 1:** IRFs for a Global Energy Price Shock: TANK (blue) vs RANK (red) under Taylor rule




- Energy price shocks induce deeper contractions in TANK than RANK.
- Demand-side effect depends on key parameters (input substitutability, price rigidity, share of HtM)

# Outline

# Conclusion

- Energy price shocks show the value of models with household heterogeneity: distributional channels affect aggregate outcomes.
- But all models are useful, pros and cons depend on the question:
  - **RANK:** tractable benchmark, clean intuition.
  - **TANK:** introduces minimal heterogeneity, useful for policy insights where aggregate distributional shifts matter, but within-group heterogeneity is not essential.
  - **HANK:** richer distributional dynamicss
- Together: a nested hierarchy for studying heterogeneity in macro.

$$\text{RANK} \subset \text{TANK} \subset \text{HANK}$$

# Modeling Choices vs. Bilbiie (2008, 2019)

- We follow Bilbiie (2019), a streamlined version of the 2008 JET model.
- **Zero profits in steady state** to sustain full-insurance baseline ($c^S = c^H$ in SS):
  - Bilbiie (2008): fixed costs in production.
  - Here (and in later work): optimal production subsidy $\Rightarrow$ steady-state markup $= 1$ and profits $= 0$.
- Constant returns to scale assumed from the outset (CRS; $\alpha = 0$ in Bilbiie (2008)'s notation).

# HtM hours and a useful knife-edge

$$\hat{H}_t^H = \frac{\varphi + \sigma^{-1}(1-\chi)}{\varphi + \sigma^{-1}} \cdot \frac{1}{\varphi}\,\hat{w}_t.$$

- If $\sigma = 1$ and $\tau^D = 0$ (so $\chi = 1 + \varphi$), then $\hat{H}_t^H = 0$: HtM hours are flat (only wages move for them).
- Useful for intuition: in that case, HtM *spending* moves through the wage, not hours; S react through intertemporal substitution.

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